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A COMPUTER PROGRAM FOR STRUCTURAL RESPONSE TO SHIP SLAMMING (SLAM)

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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md. 20084

A COMPUTER PROGRAM FOR STRUCTURAL RESPONSE TO SHIP SLAMMING (SLAM)

by

Gary P. Antonides

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SHIP ACOUSTICS DEPARTMENT
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Report 4480

The Naval Ship Research and Development Center is a U. S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland with the Marine Engineering Laboratory at Annapolis, Maryland.

Naval Ship Research and Development Center

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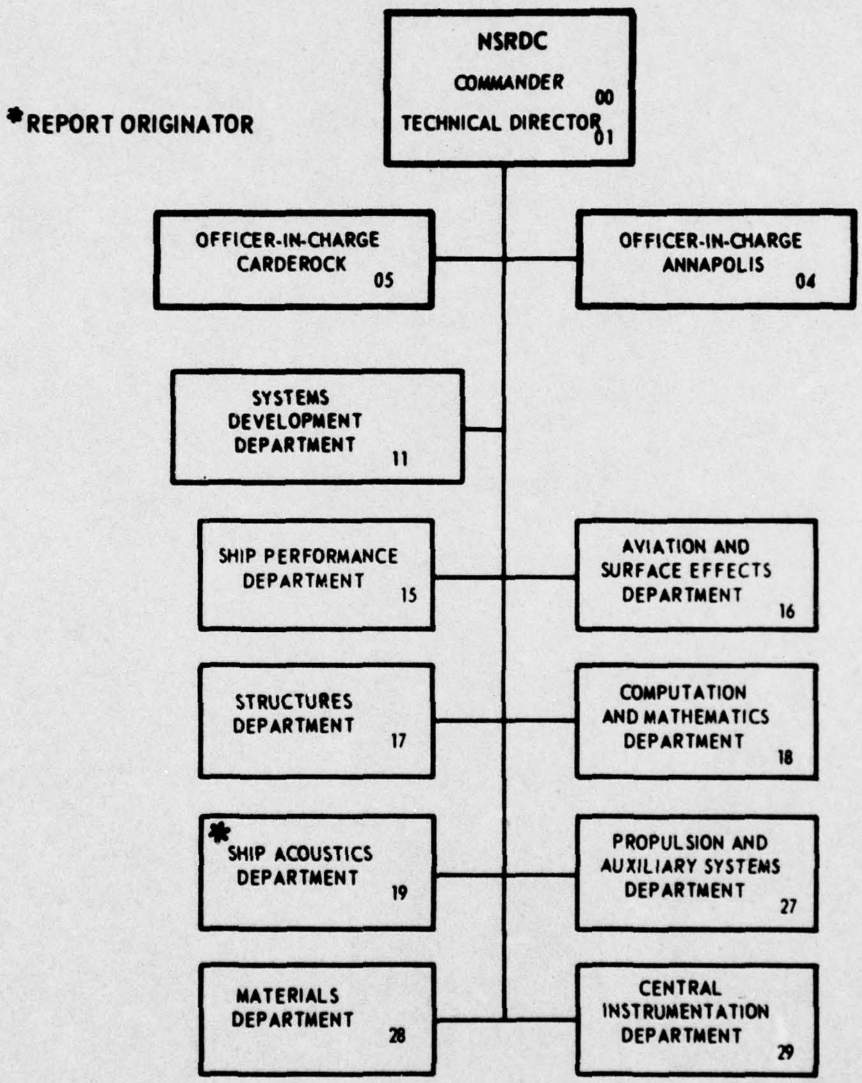

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This report describes a computer program, SLAM, which uses ship slamming-impact forces and structural parameters of the hull to calculate the vertical hull-girder vibratory response in terms of displacements, accelerations, bending moments, shear forces, and stresses. The normal mode method is used so that the user can calculate only the modes of interest and can eliminate the rigid body and higher modes. Modal responses are calculated with a over

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time-marching technique. The program uses a finite-element model of a beam which is suitable for conventional monohulls. Modifications to accommodate other model configurations can be made.

Test problems show good agreement with exact solutions for a uniform Euler beam. Sample calculations are made on a MARINER-Class hull using 3 modes and again using 10 modes.

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TABLE OF CONTENTS

LIST OF FIGURES

Page

LIST OF TABLES

ABSTRACT

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ADMINISTRATIVE INFORMATION

The work described in this report was authorized and funded by the Naval Sea Systems Command (SEA 03412) under Project S-F354 21, Task Area S-F354 21 007, Program Element 62512N, Work Units 1-1506-006 and 1-1568-102.

INTRODUCTION

BACKGROUND

The Naval Ship Research and Development Center (the Center) has developed a computer program for evaluating the impact loads and the main hull girder response associated with slamming of a ship at sea so that a more rational design of high-speed naval ships can be achieved. The program will calculate impact forces as a function of the configuration of the ship bottom, speed, draft, trim, and state of sea. Impact forces and structural parameters will then be used to obtain the hull girder response in terms of relative displacements, accelerations, bending moments, shear forces, and stresses. At present the computer program is in two parts; one calculates the forces and the other the response. They could easily be combined. Results from the overall study are presented in Reference 1.

This report presents details of the program dealing with the hull response. Hull structural characteristics and impact forces are the input to the program.

A review of experimental data about slamming shows that the hull response is most significant in the lower modes of vibration. Also, it has been observed that although the

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[1] Ochi, M. K. and L. E. Motter, "Prediction of Slamming Characteristics and Hull Responses for Ship Design," Society of Naval Architects and Marine Engineers Transactions (1973). A complete listing of references is given on page 41.

forces are dependent on rigid body motion, the rigid body motion itself is affected very little by slamming.

To determine whether the inclusion of shear rigidity and rotary inertia would increase the accuracy of the program significantly, Center reports about calculated natural frequencies of ship hulls were reviewed. Three were found that contained useful comparisons.[2-4] A summary of these results and some unpublished data are given in Table 1.

Rotary inertia is not now normally included in vibration calculations at the Center, and the example of the T-AGM-19 indicates that the practice is justified. Shear rigidity is normally included because the calculations usually involve at least the first five modes of vibration, and in the higher modes shear rigidity significantly influences the modal frequencies. Another trend which is reflected in the table, and which is to be expected, is that the shear rigidity has less influence on long slender ships than on short deep ships. Slamming is more of a problem on long slender hulls. To make the program as general as possible, however, shear rigidity was included.

It is recognized that the accuracy of natural frequency calculations is only one factor in the accuracy of hull response calculations, but it is one of the few indicators available.

Many of the requirements of this program are similar to those for the author's thesis[5] at Catholic University, and many of the same techniques are used.

APPROACH

Calculation of steady-state hull vibration at the Center has normally involved lumping the hull parameters. However, the advantages of a finite-element model are enough to warrant a different breakdown of parameters. A finite-element beam model is used.

The normal mode method seems particularly suited to the problem of ship slamming and is used in the program. It enables the user to calculate only the modes that deserve consideration and to eliminate the higher modes and rigid body motion. An alternative method would involve representing the buoyancy of the ship so as to restrain rigid body pitch and heave.

To use the normal mode approach it is necessary to calculate all of the natural frequencies and mode shapes of interest. The next major step is to transform the coordinates

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[2] Robinson, D. C., "Calculated Natural Frequencies and Normal Modes of the Guided Missile Cruiser USS LONG BEACH (CG(N)-9)," David Taylor Model Basin Report 2100 (Jan 1966).

[3] Perkins, R. L., "Calculated Natural Frequencies and Normal Modes of Vibration on Range Instrumentation Ship (T-AGM-19)," David Taylor Model Basin Report 1997 (Jun 1965).

[4] McGoldrick, R. T. and V. L. Russo, "Hull Vibration Investigation on SS GOPHER MARINER," David Taylor Model Basin Report 1060 (Jul 1956).

[5] Antonides, G. P., "A Computer Program for Normal Mode Solutions in Structural Dynamics," Master's Thesis at Catholic University, Washington, D. C. (Dec 1970).

TABLE 1 – CALCULATED VERTICAL BENDING NATURAL FREQUENCIES OF SHIP HULLS WITH AND WITHOUT SHEAR RIGIDITY AND ROTARY INERTIA

Ship (Reference)	Length feet	Depth feet	Length/Depth Ratio	Mode	Calculated Natural Frequency, Hz			Measured Frequency, Hz
					Shear, Bending, Rotary Inertia	Shear, Bending	Bending	
USS LONG BEACH (CGN-9)[2]	721.0	45.0	15.3	1		0.83	0.86	1.0
				2		1.73	1.93	2.0
				3		2.80	3.39	3.0
				4		3.98	5.16	4.5
T-AGM-19[3]	575.0	47.0	12.24	1	0.99	1.00	1.08	
				2	2.16	2.19	2.70	
				3	3.40	3.46	4.82	
				4	4.76	4.83	7.42	
MARINER Class[4] (And Unpublished Data)	528.0	44.5	11.8	1		1.26	1.38	1.34
				2		2.54	3.29	2.57
				3		3.89	5.90	3.46

to obtain the uncoupled equations of motion. Each equation then represents a mode of vibration. The modal equations are solved for the modal displacements in the modes of interest, and these are transformed back into physical displacements. The accelerations, bending moments, shear forces, and stresses can then be found from the displacements and ship parameters.

For the solution of modal equations, the program uses a time-marching method involving finite differences with respect to time. The modal displacements at any instant are calculated as a function of the displacements at earlier times, system parameters, and applied forces. To minimize the use of computer storage, all of the required quantities are calculated for one instant of time and then printed before proceeding to the next time increment. Only the quantities required for the next calculation are stored.

In the following sections of this report the procedure is developed analytically, the equations used directly in the computer program are indicated, the program itself is described, and the program is evaluated with a series of test problems.

MATHEMATICAL MODEL FOR SHIP HULL

STRUCTURAL MODEL

The ship hull is considered as a nonuniform beam divided into 20 or less equal sections or elements. Nodes or coordinates are used at the ends of each section as shown in Figure 1. The mass m, bending rigidity EI/ℓ, and shear rigidity KAG/ℓ of the sections are numbered as shown in Figure 1. The deflections, y_i, are taken at the nodes at the ends of the sections. The slamming forces F_i are considered as discrete forces acting vertically at the nodes. Both deflections and forces are positive upwards. Damping characteristics are discussed later.

UNDAMPED EQUATIONS OF MOTION

The undamped equations of motion in matrix form are:

$$[M]\ \{\ddot{y}\} + [K]\ \{y\} = \{F\} \tag{1}$$

where $[M]$ is the mass matrix (including added mass due to water)

$[K]$ is the stiffness matrix

$\{\ddot{y}\}$ is the vector of vertical accelerations

$\{y\}$ is the vector of vertical deflections

$\{F\}$ is the vector of applied forces

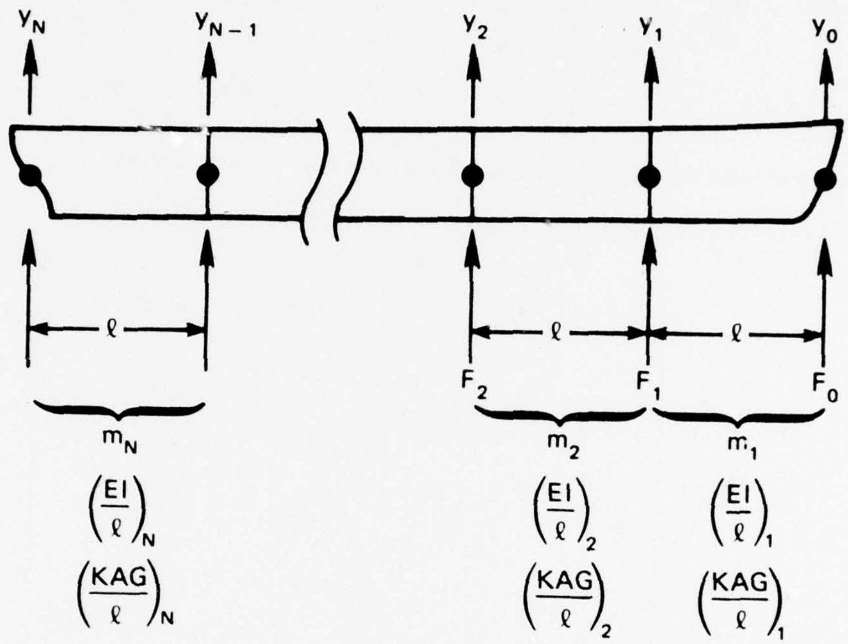

Figure 1 – Structural Model for Vertical Vibration of a Ship Hull

The mass and stiffness matrices are based on a finite-element beam model and are taken from Reference 6. The mass matrix for a single finite element between nodes i and j, referred to the coordinate set $\{ y_i, \theta_i, y_j, \theta_j \}$ where θ is slope, is

$$[M] = \frac{m\ell}{420} \begin{bmatrix} 156 & -22\ell & 54 & 13\ell \\ -22\ell & 4\ell^2 & -13\ell & -3\ell^2 \\ 54 & -13\ell & 156 & 22\ell \\ 13\ell & -3\ell^2 & 22\ell & 4\ell^2 \end{bmatrix} \tag{2}$$

where m is the mass of the element, and ℓ is the length of the element.

The stiffness matrix for the element, referred to the same coordinate set, is

$$[K] = \begin{bmatrix} R & -\frac{\ell}{2} R & -R & \frac{\ell}{2} R \\ -\frac{\ell}{2} R & \frac{\ell^2}{4} R + \frac{EI}{\ell} & \frac{\ell}{2} R & \frac{\ell^2}{4} R - \frac{EI}{\ell} \\ -R & \frac{\ell}{2} R & R & \frac{\ell}{2} R \\ \frac{\ell}{2} R & \frac{\ell^2}{4} R - \frac{EI}{\ell} & \frac{\ell}{2} R & \frac{\ell^2}{4} R + \frac{EI}{\ell} \end{bmatrix} \tag{3}$$

where

$$R = \left(\frac{\ell}{KAG} + \frac{\ell^3}{12\ EI} \right)^{-1} \tag{4}$$

The mass and stiffness matrices for the entire beam are assembled by superposition of the element matrices. The resulting matrices for a beam with M elements and M + 1 nodes will be of the order N = 2M + 2.

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[6] MacNeal, Richard H., "The NASTRAN Theoretical Manual (Level 15)," National Aeronautics and Space Administration SP-221(01), Washington, D. C. (Apr 1972).

DAMPING MATRIX

To include damping in the equations of motion, consider a viscous damping matrix [C] so that the equations of motion become

$$[M]\ \{\ddot{y}\} + [C]\ \{\dot{y}\} + [K]\ \{y\} = \{F\} \tag{5}$$

The normal mode method requires that these equations be uncoupled, and this can only be done if the damping matrix is proportional to either the mass matrix, or the stiffness matrix, or a linear combination; see Reference 7 or section about uncoupling equations of motion.

$$[C] = a\ [M] + b\ [K] \tag{6}$$

This requirement is broad enough to accommodate the types of damping normally used with hull-vibration damping. Reference 8 considers two types of damping coefficients c, (1) Rayleigh damping in which c/μ is a constant C_R where $\mu = m/\ell$ is the mass per unit length, and (2) frequency dependent damping in which $c/\mu\omega$ is a constant C_F where ω is the circular frequency. Both of these are mass proportional and can be expressed in terms of [M]. For Rayleigh damping

$$[C] = (C_R/\ell)\ [M], \quad a = C_R/\ell, \quad b = 0$$

For frequency dependent damping it is more complicated, since the ω varies. In this case, we use the factor C_F/ℓ but after uncoupling the modes, each modal damping constant is multiplied by the natural frequency of that mode; see section about uncoupling equations of motion.

Experimental data (vibration generator tests and anchor drops) cited in Reference 8 indicate that frequency dependent damping is more appropriate and that the value $C_F = 0.03$ is an average value for several ships tested.

Physically mass proportional damping corresponds to dampers connected between the nodes and an inertial reference.

A portion of the total hull damping must be due to hysterisis which would be represented by dampers working against the relative rotational velocities (changes in slope) and shear velocities of adjacent elements. Although this type of damping has not been used in many

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[7] Hurty, W. C. and M. F. Rubinstein, "Dynamics of Structures," Prentice-Hall, Inc., Englewood Cliffs, N. J. (1964).

[8] McGoldrick, R. T., "Ship Vibration," David Taylor Model Basin Report 1451 (Dec 1960).

calculations, further development may justify its use, and it is included in the program. The moment and shear transmitted between two sections by the bending rigidity EI and shear rigidity KAG is dependent on the slopes and deflections of adjacent elements. If a damper also transmits a moment and shear between two elements (but as a function of the time rate of change of slopes and deflections) then the damping matrix must take the same form as the stiffness matrix. If, in addition, the values of the coefficients are proportional to the rigidities, then the damping matrix can be written

$$[C] = b\,[K]$$

NATURAL FREQUENCIES AND MODE SHAPES

MODES OF VIBRATION OF A FREE-FREE BEAM

The first step in solving the equations of motion of the idealized beam is the solution of the free, undamped vibration problem.

$$[M]\ \{\ddot{y}\} + [K]\ \{y\} = \{0\} \tag{7}$$

We seek solutions of the form

$$\{y\} = \{X\}\ e^{i\omega_n t} \tag{8}$$

Equation (7) becomes

$$-\omega_n^2\ [M]\ \{X\} + [K]\ \{X\} = \{0\} \tag{9}$$

There are as many solutions for ω_n as there are degrees of freedom. Each value of ω_n is an eigenvalue (natural frequency) of the system, and is dependent only on the masses and stiffnesses of the system. For each eigenvalue there is a corresponding vector $\{X\}$ which, together with the eigenvalue, satisfies Equation (9). The vector $\{X\}$ is an eigenvector and represents the mode shape. The mode shapes can be determined only to within a multiplicative constant. The matrix $[X]$ formed by the column vectors $\{X\}$ is called the eigenvector matrix.

If the constraints are such that rigid body motion is possible, $\omega_n = 0$ will be a solution with a multiplicity corresponding to the number of rigid body modes. A free-free beam considered in the vertical direction has two rigid body modes, one in translation and one in rotation.

The two-noded mode will be the first of the remaining modes, the three-noded mode will be the second, etc.

DETERMINANT METHOD OF SOLUTION

To solve Equation (9), we can combine the mass and stiffness properties into one operator matrix.

$$([K] - \omega_n^2 [M]) \{X\} = \{0\} \tag{10}$$

This represents a set of N homogeneous algebraic equations in X. For $\{X\}$ to have non-zero solutions, the determinant of the operator matrix must be zero.

$$|[K] - \omega_n^2 [M]| = 0 \tag{11}$$

Trial values of ω_n are substituted into Equation (11), and, by interpolation, the values of ω_n that cause the determinant to be zero are found; they are the natural circular frequencies of the system.

To obtain the eigenvectors, the ω_n are substituted back into Equation (10), and the vector $\{X\}$ is found for each ω_n. If we let

$$[D] = ([K] - \omega_n^2 [M])$$

then Equation (10) can be written

$$[D] \{X\} = \{0\}$$

which when expanded takes the form

$$\begin{array}{c} D_{11} X_1 + D_{12} X_2 + \ldots\ldots + D_{1N} X_N = 0 \\ D_{21} X_1 + D_{22} X_2 + \ldots\ldots + D_{2N} X_N = 0 \\ \vdots \qquad\qquad\qquad \vdots \\ D_{N1} X_1 + D_{N2} X_2 + \ldots\ldots + D_{NN} X_N = 0 \end{array} \tag{12}$$

Since mode shapes can only be determined within multiplicative constants we can specify any one X_i in each mode. We find it convenient to let $X_N = 1$ in all modes. Then the first $N - 1$ equations are written

$$
\begin{array}{lllll}
D_{11} X_1 & + D_{12} X_2 & + \ldots\ldots\ldots + D_{1,N-1} X_{N-1} & = -D_{1N} \\
D_{21} X_1 & + D_{22} X_2 & + \ldots\ldots\ldots + D_{2,N-1} X_{N-1} & = -D_{2N} \\
 & \vdots & & \vdots \\
D_{N-1,1} X_1 & + D_{N-1,2} X_2 & + \ldots\ldots\ldots + D_{N-1,N-1} X_{N-1} & = -D_{N-1,N}
\end{array}
\tag{13}
$$

These are solved as a set of nonhomogeneous algebraic equations, and, after the vectors $\{X\}$ are found, each can be substituted into the N*th* equation to check

$$D_{N1} X_1 + D_{N2} X_2 + \ldots\ldots\ldots + D_{N,N-1} X_{N-1} + D_{NN} = 0$$

This "determinant method" has several advantages over other commonly used methods for finding eigenvalues and eigenvectors. Possibly the greatest advantage is that it is not necessary to reduce the order of the dynamic matrix by the number of rigid body modes, thereby obtaining the equations of motion in generalized coordinates. While this may not be difficult for the beam, if the program is later adapted to other transient problems, each type of constraint must be treated separately.

Other advantages are that it is more accurate for high modes than some other methods and that only the modes desired need be calculated.

CALCULATION OF RESPONSE

ORTHOGONALITY

In this section we will show that the eigenvectors are orthogonal with respect to the matrices [M] and [K].

If we write Equation (9) for the i*th* mode, we have

$$\omega_i^2 \, [M] \left\{ X^{(i)} \right\} = [K] \left\{ X^{(i)} \right\} \tag{14}$$

where $\{X^{(i)}\}$ is the mode shape (eigenvector) for the *ith* mode. Premultiplying this by $\{X^{(j)}\}^T$, the transposed eigenvector for a different mode, we get

$$\omega_i^2 \{X^{(j)}\}^T [M] \{X^{(i)}\} = \{X^{(j)}\}^T [K] \{X^{(i)}\} \tag{15}$$

Next, take the transpose of both sides of Equation (15) and apply the rule which states that the transpose of a matrix product is equal to the product of the transposed matrices in reverse order.

$$\omega_i^2 \{X^{(i)}\}^T [M] \{X^{(j)}\} = \{X^{(i)}\}^T [K] \{X^{(j)}\} \tag{16}$$

We have used the fact that [M] and [K] are symmetric and therefore equal to their transposes. Now we write an equation identical to Equation (15) except interchanging indices.

$$\omega_j^2 \{X^{(i)}\}^T [M] \{X^{(j)}\} = \{X^{(i)}\}^T [K] \{X^{(j)}\} \tag{17}$$

Subtract Equation (17) from Equation (16)

$$(\omega_i^2 - \omega_j^2) \{X^{(i)}\}^T [M] \{X^{(j)}\} = \{0\} \tag{18}$$

For $\omega_i \neq \omega_j$

$$\{X^{(i)}\}^T [M] \{X^{(j)}\} = \{0\} \tag{19}$$

Also, from Equation (17) we have

$$\{X^{(i)}\}^T [K] \{X^{(j)}\} = \{0\} \tag{20}$$

Equations (19) and (20) reflect the definition of orthogonality with respect to a weighting matrix, in this case [M] or [K]. Note that if two natural frequencies are the same, their mode shapes are not necessarily orthogonal.

NORMALIZATION

We mentioned before that an eigenvector represents a characteristic pattern of relative displacements associated with a particular mode of vibration. This means that we can multiply all the components of an eigenvector by any constant, and it will still represent the same mode shape. We are able to use this fact to simplify our analysis.

Premultiplying Equation (14) by $\{X^{(i)}\}^T$, the transposed eigenvector for the *ith* mode, we obtain

$$\omega_i^2 \{X^{(i)}\}^T [M] \{X^{(i)}\} = \{X^{(i)}\}^T [K] \{X^{(i)}\} \tag{21}$$

First we will consider the triple matrix product on the left side. It can be shown that the product will be some constant, say m_i

$$\{X^{(i)}\}^T [M] \{X^{(i)}\} = m_i \tag{22}$$

We wish to normalize the vector $\{X^{(i)}\}$ and its transpose by multiplying it by a constant n_i so that m_i becomes unity

$$n_i^2 \{X^{(i)}\}^T [M] \{X^{(i)}\} = 1 \tag{23}$$

Written in subscript notation

$$n_i^2 \sum_j \sum_k m_{jk} X_{ji} X_{ki} = 1 \tag{24}$$

where m_{jk} is an element of $[M]$, and X_{ji} is the *jth* component of $\{X^{(i)}\}$. Solving for n_i

$$n_i = \frac{1}{\sqrt{\sum_j \sum_k m_{jk} X_{ji} X_{ki}}} \tag{25}$$

Let the normalized eigenvector be denoted by $\{X_n^{(i)}\}$, so that

$$\{X_n^{(i)}\} = n_i \{X^{(i)}\} \tag{26}$$

If we write Equation (21) using our normalized eigenvectors, we get

$$\omega_i^2 \{X_n^{(i)}\}^T [M] \{X_n^{(i)}\} = \{X_n^{(i)}\}^T [K] \{X_n^{(i)}\} \tag{27}$$

We have shown that

$$\{X_n^{(i)}\}^T [M] \{X_n^{(i)}\} = 1 \tag{28}$$

which means t

$$\{X_n^{(i)}\}^T [K] \{X_n^{(i)}\} = \omega_i^2 \tag{29}$$

If we assemble the normalized eigenvector matrix $[X_n]$, we can show by use of Equations (19), (20), (28), and (29) that

$$[X_n]^T [M] [X_n] = [I] \tag{30}$$

$$[X_n]^T [K] [X_n] = [\omega^2] \tag{31}$$

where $[\omega^2]$ is a diagonal matrix of the squares of ω_i. The last two equations are useful in our normal mode analysis.

The eigenvector matrix for the free-free beam is obtained by assembling the (N – 2) and the rigid body mode shapes into an (N x N) matrix.

UNCOUPLING THE EQUATIONS OF MOTION

Our next task is to uncouple the equations of motion so that we have a series of equations (one for each mode) resembling that of a simple oscillator. We start with Equation (5)

$$[M] \{\ddot{y}\} + [C] \{\dot{y}\} + [K] \{y\} = \{F\} \tag{5}$$

where $\{y\}$ and its derivatives and $\{F\}$ are functions of time. The displacement response may be expressed as a superposition of normal modes

$$\{y(t)\} = [X_n] \{q(t)\} \tag{32}$$

where $\{q\}$ is a vector of time-dependent generalized coordinates. They are called normal coordinates since they are associated with the normalized modal matrix $[X_n]$. Substitution gives

$$[M]\,[X_n]\,\{\ddot{q}\} + [C]\,[X_n]\,\{\dot{q}\} + [K]\,[X_n]\,\{q\} = \{F\} \tag{33}$$

Premultiply this equation by $[X_n]^T$

$$[X_n]^T\,[M]\,[X_n]\,\{\ddot{q}\} + [X_n]^T\,[C]\,[X_n]\,\{\dot{q}\} + \{X_n\}^T\,[K]\,[X_n]\,\{q\} = [X_n]^T\,\{F\} \tag{34}$$

Applying Equations (30), (31), and (6) yields

$$[I]\,\{\ddot{q}\} + a\,[I]\,\{\dot{q}\} + b\,[\omega^2]\,\{\dot{q}\} + [\omega^2]\,\{q\} = [X_n]^T\,\{F\} \tag{35}$$

If we let

$$\{P(t)\} = [X_n]^T\,\{F(t)\} \tag{36}$$

$$[G] = a\,[I] + b\,[\omega^2] \tag{37}$$

Equation (35) becomes

$$\{\ddot{q}\} + [G]\,\{\dot{q}\} + [\omega^2]\,\{q\} = \{P(t)\} \tag{38}$$

The matrices $[G]$ and $[\omega^2]$ are diagonal so that Equation (38) can be written as a system of uncoupled equations

$$\ddot{q}_i(t) + G_i\dot{q}_i(t) + \omega_i^2\,q_i(t) = P_i(t),\ i = 1, 2, \ldots N \tag{39}$$

where

$$G_i = a + b\omega_i^2 \tag{40}$$

Equations (39) have the form of the equations of motion of damped simple oscillators, and each equation represents a particular mode of vibration. The solution of Equations (39)

is well known, and there are several methods that could be used. Our computer program uses a time-marching method.

TIME-MARCHING METHOD

There are several time-maching methods which approximate the differential equations of motion with finite-difference equations so that the displacement at a particular time t_{n+1} is an algebraic function of the displacement at time t_n, the parameters of the system, and the applied force. These methods appear simpler than transform methods which involve a large amount of numerical integration.

The particular time-marching method used herein is one developed by Chan, Cox, and Benfield.[9] In this method we consider time to be divided into many small, equal time increments, h, where $h = t_{n+1} - t_n = t_n - t_{n-1}$. The modal displacements q_{n+1} at the time t_{n+1} are calculated, then transformed back to physical coordinates.

The equations are derived in Reference 9; only the results are stated here. The main recurrence formula for the solution of Equation (39) is

$$Zq_{n+1} = Aq_n - Wq_{n-1} + \beta h^2 P_{n+1} + (1 - 2\beta) h^2 P_n + \beta h^2 P_{n-1}, \; n > 0 \qquad (41)$$

where

$$Z = 1 + (h/2)G + \beta h^2 \omega^2$$

$$A = 2 - (1 - 2\beta) h^2 \omega^2 \qquad (42)$$

$$W = 1 - (h/2)G + \beta h^2 \omega^2$$

Equation (41) must be applied to each of the modal displacements; however, modal subscripts have been omitted to avoid confusion with the time subscripts. The quantity β can be assigned any value between 0 and 1/4. Different values of β correspond to different approximating assumptions about the acceleration between time steps (such as a constant acceleration within each increment or a linear function). The effect of β on accuracy is discussed in the Evaluation.

Equations (41) and (42) are not valid for the first time increment. The following equation, which includes initial conditions must be used.

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[9] Chan, S. et al., "Transient Analysis of Forced Vibrations of Complex Structural-Mechanical Systems," Journal of the Royal Aeronautical Society, Vol. 66, No. 457 (Jul 1962).

$$Zq_i = Qq_0 + R\dot{q}_0 + SP_0 + \beta h^2 P_1 \tag{43}$$

where

$$Q = 1 + hG/2 - (1/2 - \beta)\, h^2 \omega^2 - (1/4 - \beta)\, h^3 G\omega^2$$

$$R = h - (1/4 - \beta)\, h^3 G^2 \tag{44}$$

$$S = (1/2 - \beta)\, h^2 + (1/4 - \beta)\, h^3 G$$

q_0 = initial modal displacement

$\dot{q}_0$ = initial modal velocity

After modal displacements at a particular time have been determined by the time-marching equations, the nodal displacements are obtained for Equation (32).

$$y = [X_n] \quad q \tag{32}$$

From the nodal displacements, we can obtain the velocities and accelerations. The velocity is found as a second order approximation of the slope of the displacement, and the acceleration is calculated from the slope of the velocity or curvature of displacement

$$\dot{X}_{n+1} = \frac{1}{2h} (X_{n-1} - 4X_n + 3X_{n+1}) \tag{45}$$

$$\ddot{X}_{n+1} = \frac{1}{h^2} (X_{n-1} - 2X_n + X_{n+1}) \tag{46}$$

The moment in the element between nodes i and j is approximated by

$$M_{ij} = \frac{EI_{ij}}{\ell} (\theta_j - \theta_i) \tag{47}$$

The stress $S_{b_{ij}}$ due to bending in the element between nodes i and j is given by

$$S_{b_{ij}} = M_{ij}\, c/I_{ij} \tag{48}$$

where c is the vertical distance from the neutral axis to the point where the stress is being calculated.

The shear force V_j at the jth node is

$$V_j = \frac{M_{ij} - M_{jk}}{\ell} \tag{49}$$

The shear stress at any station near the neutral bending axis is

$$S_{S_i} = \frac{V_i}{KA} \tag{50}$$

where KA is the effective area at the station. (In our model the effective areas are given by elements, so the KA of either adjacent element could be used.)

DESCRIPTION OF PROGRAM

SUMMARY OF APPLICABLE EQUATIONS

Before discussing the details of the program, this section lists the equations used in the program from the preceding analysis and indicates briefly what steps are necessary.

First the mass and stiffness matrices are obtained from the superposition of the individual element matrices, Equations (2) and (3),

$$[M] = \frac{m\ell}{420}\begin{bmatrix} 156 & -22\ell & 54 & 13\ell \\ -22\ell & 4\ell^2 & -13\ell & -3\ell^2 \\ 54 & -13\ell & 156 & 22\ell \\ 13\ell & -3\ell^2 & 22\ell & 4\ell^2 \end{bmatrix} \tag{2}$$

$$[K] = \begin{bmatrix} R & -\frac{\ell}{2} R & -R & \frac{\ell}{2} R \\ -\frac{\ell}{2} R & \frac{\ell^2}{4} R + \frac{EI}{\ell} & \frac{\ell}{2} R & \frac{\ell^2}{4} R - \frac{EI}{\ell} \\ -R & \frac{\ell}{2} R & R & \frac{\ell}{2} R \\ \frac{\ell}{2} R & \frac{\ell^2}{4} R - \frac{EI}{\ell} & \frac{\ell}{2} R & \frac{\ell^2}{4} R + \frac{EI}{\ell} \end{bmatrix} \tag{3}$$

$$R = \left(\frac{\ell}{KAG} + \frac{\ell^3}{12\ EI} \right)^{-1} \tag{4}$$

Then solutions are obtained for natural frequencies ω_n using Equation (11)

$$|[K] - \omega_n^2 [M]| = 0 \tag{11}$$

The eigenvectors can then be found using Equation (10)

$$([K] - \omega_n^2 [M]) \{X\} = 0 \tag{10}$$

where $X_N = 1$.

Then the eigenvector matrix must be normalized by means of the following two equations

$$n_i = \frac{1}{\sqrt{\sum_j \sum_k m_{jk} X_{ji} X_{ki}}} \tag{25}$$

$$\{X_n^{(i)}\} = n_i \{X^{(i)}\} \tag{26}$$

The nodal exciting forces must be transformed into modal exciting forces

$$\{P\} = [X_n]^T \{F\} \tag{36}$$

The modal damping is calculated from

$$G_i = a + b\omega_i^2 \tag{40}$$

At this point we have all we need to start our time-marching procedure. First we note that the starting Equation (43) and the main recurrence Equation (41) require the calculation of the following parameters

$$\begin{aligned} Z &= 1 + (h/2)G + \beta h^2 \omega^2 \\ A &= 2 - (1 - 2\beta)\, h^2 \omega^2 \\ W &= 1 - (h/2)G + \beta h^2 \omega^2 \end{aligned} \tag{42}$$

$$\begin{aligned} Q &= 1 + hG/2 - (1/2 - \beta)\, h^2 \omega^2 - (1/4 - \beta)\, h^3 \omega^2 G \\ R &= h - (1/4 - \beta)\, h^3 G^2 \\ S &= (1/2 - \beta)\, h^2 + (1/4 - \beta)\, h^3 G \end{aligned} \tag{44}$$

Although we have omitted modal subscripts, these parameters must be calculated for each mode.

Next we start a series of calculations which must be performed at each time increment. The first step is either the starting equation

$$Zq_i = Qq_0 + R\dot{q}_0 + SP_0 + \beta h^2 P_1 \tag{43}$$

or the main recurrence formula

$$Zq_{n+1} = Aq_n - Wq_{n-1} + \beta h^2 P_{n+1} + (1 - 2\beta)\, h^2 P_n + h^2 P_{n-1} \tag{41}$$

After the modal displacements at time t_{n+1} have been calculated, we calculate the physical or nodal displacements for time t_{n+1} using

$$\{y\} = [X_n]\{q\} \tag{32}$$

The nodal velocity and acceleration are calculated from

$$\dot{X}_{n+1} = \frac{1}{2h}(X_{n-1} - 4X_n + 3X_{n+1}) \tag{45}$$

$$\ddot{X}_{n+1} = (1/h^2)(X_{n-1} - 2X_n + X_{n+1}) \tag{46}$$

The bending moments are calculated from

$$M_{ij} = \frac{EI_{ij}}{\ell}(\theta_j - \theta_i) \tag{47}$$

The bending stresses are calculated from

$$S_{b_{ij}} = M_{ij}\, c/I_{ij} \tag{48}$$

The shear force is calculated from Equation (49) and the shear stress from Equation (50)

$$V_j = \frac{M_{ij} - M_{jk}}{\ell} \tag{49}$$

$$S_{S_i} = \frac{V_i}{KA} \tag{50}$$

After these calculations the output for each desired station for each time increment is printed, and the program begins the loop again for the next time increment.

MAIN PROGRAM

Appendix A is a flow chart of the program SLAM and indicates how the equations of the previous section are incorporated into the program. SLAM is designed to calculate the vertical transient response of a beam-type structure with up to 20 elements. Vertical forces

can be applied to the first 10 stations. At each of the stations the force can be specified by as many as 100 points with respect to time. The response is calculated for every hundredth of a second which is adequate to describe motions with frequencies at least up to 10 hertz. Output for every increment, or for every NPRINT increments, can be printed as desired. The user specifies the number of modes to be included in the response calculation and can opt to have the natural frequencies and mode shapes printed out. At each station for which output is requested, the displacement, acceleration, bending moment, shear force, and bending and shear stresses are printed out. Since the output is printed by stations and the bending moment, bending stress, and shear stress are associated with the elements, those quantities are given for the element just forward and just aft of the station named. SLAM will account for any of three types of damping:

1. Mass proportional
2. Stiffness proportional
3. Mass and frequency proportional

For the input, any consistent set of units can be used, except that the value of E used can be a different set of units. The stresses calculated will be in that set of units.

SUBROUTINES

FORCES – This subroutine takes the forces as plotted for the program input, interpolates where necessary, and calculates the modal forces which are used in the main program.

MATINS – This subroutine is in the Center library of subroutines and can either find the determinant of a square matrix [A] or indicate if it is singular and either find the inverse of the matrix [A] or solve a set of simultaneous linear equations of the form [A] {X} = [B]. In SLAM it is used first to find determinants in the natural frequency part of the program. Then it is used to solve simultaneous equations in obtaining the mode shapes.

INPUT/OUTPUT

Sample input for SLAM is given in Figure 2. The necessary format and description of the input is given in Table 2.

Sample output for SLAM is given in Figure 3.

EVALUATION

ANALYTICAL APPROACH

The analytics used in SLAM were selected specifically for the ship-slamming application; therefore, they should provide maximum flexibility and utility.

```
20 03 01 01 01 01 71 07 04 02
 26.25      0.0        0.0        0.03        5.00        30.0E+06  11.9E+06
 7.000E+00 4.400E+09 5.300E+06 28.5
 1.230E+01 5.800E+09 5.200E+06 28.5
 1.600E+01 7.430E+09 4.760E+06 28.5
 1.640E+01 8.730E+09 4.400E+06 28.5
 2.200E+01 1.040E+10 4.140E+06 28.5
 3.000E+01 1.203E+10 3.900E+06 28.5
 4.220E+01 1.340E+10 3.730E+06 28.5
 5.250E+01 1.428E+10 3.570E+06 28.5
 6.100E+01 1.482E+10 3.440E+06 28.5
 7.150E+01 1.500E+10 3.400E+06 28.5
 7.420E+01 1.498E+10 3.450E+06 28.5
 7.180E+01 1.476E+10 3.630E+06 28.5
 6.580E+01 1.426E+10 3.920E+06 28.5
 5.880E+01 1.352E+10 4.320E+06 28.5
 5.800E+01 1.254E+10 4.640E+06 28.5
 5.460E+01 1.130E+10 4.600E+06 28.5
 3.830E+01 9.840E+09 3.890E+06 28.5
 2.460E+01 8.100E+09 3.670E+06 28.5
 1.500E+01 5.960E+09 3.070E+06 28.5
 7.000E+00 4.560E+09 2.320E+06 28.5
       0.      0.
       .01    18.03      18.03
       .02    50.65      50.65
       .03    88.7       88.7
       .04    129.1      129.1
       .05    170.7      170.7
       .06    196.95     214.98     18.03
       .07    206.6      257.25     50.65
       .08    210.95     299.65     88.7
       .09    212.6      341.7      129.1
       .1     213.05     383.75     170.7
       .11    195.05     422.05     227.0
       .12    162.4      447.15     284.75
       .13    124.4      470.35     345.95
       .14    84.        493.1      409.1
       .15    42.4       515.75     473.35
       .16    16.15      509.75     523.67     30.07
       .17    6.48       496.38     558.05     78.15
       .18    2.13       453.28     586.1      134.95
       .19    .47        415.67     611.65     196.45
       .2     .06        375.71     636.       260.35
       .21      0.       319.85     666.35     346.5
       .22    0.         262.15     694.35     432.2
       .23     0.        201.       718.5      517.5
       .24    0.         137.85     741.85     604.
       .25      0.       73.55      764.05     690.5
       .26    0.         35.3       739.3      751.96     47.96
       .27     0.        16.35      693.35     791.75     114.75
       .28    0.         7.1        638.1      822.       191.
       .29    0.         2.59       578.03     848.05     272.55
       .3     0.         .52        514.02     870.7      357.2
       .31     0.        0.06       428.31     894.       465.75
       .32    0.         0.         342.55     915.55     573.
       .33    0.         0.         257.55     936.55      679.
       .34    0.         0.          171.       954.5     783.5
       .35    0.          0.        84.25      971.75      887.5
       .36    0.         0.         40.8       926.8      951.55     65.55
       .37    0.         0.         19.74       858.74    991.0      152.
       .38    0.         0.         8.65       783.65     1021.85    245.85
       .39    0.          0.        3.03       705.03     1047.4     345.4
       .4     0.         0.          .7        623.2      1069.7     447.2
       .41     0.        0.         .04        516.04      1064.5     548.5
       .42     0.         0.          0.       409.6      1039.6     630.
       .43    0.         0.         0.         303.75     1015.75    712.0
       .44    0.         0.         0.         199.55     994.05     794.5
       .45    0.         0.         0.         95.45      970.95      875.5
       .46    0.         0.         0.         48.82      892.32     893.65     50.15
       .47    0.         0.         0.         29.35      800.84     889.05     117.55
       .48    0.         0.         0.         16.76      703.76     874.3      187.3
       .49    0.         0.         0.         8.2        603.7      856.35     260.85
       .5     0.         0.         0.         3.23       501.93     835.6      336.9
       .51     0.        0.         0.         1.11       407.41     774.6      368.3
       .52    0.         0.         0.         .11        321.46     702.9      381.55
       .53     0.        0.         0.         .01        241.01      633.      392.0
       .54     0.         0.        0.         0.         158.7      557.9      399.
       .55    0.         0.         0.         0.         78.05      482.       403.95
       .56    0.         0.         0.         0.         44.54      402.04      357.5
       .57    0.         0.         0.         0.         30.27      327.27     297.0
       .58    0.         0.         0.         0.         19.74      243.98     224.25
       .59    0.         0.          0.         0.        12.5       163.       150.5
       .6     0.         0.         0.         0.          7.71       82.54      74.9
       .61    0.         0.         0.         0.         4.07       47.49       43.42
       .62     0.        0.         0.         0.         2.18       32.33      30.16
       .63    0.         0.         0.          0.         .66       20.38      19.72
       .64    0.         0.          0.         0.         .32       12.83      12.5
       .65    0.          0.         0.         0.        0.         7.74       7.74
       .66    0.         0.         0.         0.         0.         4.07       4.07
       .67    0.         0.          0.         0.         0.         2.19      2.18
       .68    0.         0.         0.         0.         0.         .66        .66
       .69    0.         0.         0.         0.         0.         .3          0.3
      5.05    0.         0.         0.         0.         0.         0.         0.
05 07 10 15
```

Figure 2 – Sample Input for SLAM (See Table 2)

TABLE 2 – EXPLANATION OF SLAM INPUT

Line Number	Format	Description
1	(I2,20(1X,I2)	N – Number of sections (20)* NMODE – Number of modes to be calculated (10)* IOPT(1) – For input IOPT(2) – For natural frequencies IOPT(3) – For mode shapes IOPT(4) – For transient response (IOPT(I) indicates what output is desired–1 if desired, 0 if not.) NPTS – Number of points in the force versus time plot including t = 0.(100)* NFOR – Number of sections to which force is applied. (10)* NOUT – Number of sections for which output is desired. (20)* NPRINT – Number of calculations for each printout.
2	(8E10.3)	DELX – Length of hull sections DAMPM – Mass-proportional damping constant DAMPK – Stiffness-proportional damping constant DAMPF – Mass and frequency proportional damping constant TSTOP – Time of last desired response E – Modulus of elasticity S – Shear modulus
3–22	(8E10.3)	EMASS(I) – Mass of hull elements EEI(I) – EI of hull elements EKAG(I) – KAG of hull elements DIST(I) – Distance from neutral bending axis to desired stress location (Consists of N lines starting at bow.)
23–93	(8E10.3)	TPT(I) – Time of each force versus time point FPT(I,J) – Force on the NFOR sections (Consists of NPTS lines; each one gives the time and NFOR forces applied at that time. Includes t = 0 at which time all forces must equal zero; also, must give forces up to t = TSTOP.
94	(I2,20(1X,I2)	LOUT(I) – Station numbers for which output is desired (Consists of NOUT station numbers. Do not include 0 and N which printout automatically.)

*Indicates maximum number allowed.

Figure 3 – Sample Output for SLAM

N	NODE	DELX	DAMPM	DAMPX	DAMPF	TSTOP	E	G	NPRINT
20	3	2.63E+01	0.	0.	3.00E-02	5.00E+00	3.00E+07	1.15E+07	2

STA		MASS	EI	KAG	DIST
0	1	7.000E+00	4.490E+09	5.100E+06	2.850E+01
1	2	1.230E+01	5.990E+09	5.200E+06	2.850E+01
2	3	1.600E+01	7.410E+09	4.760E+06	2.850E+01
3	4	1.640E+01	8.730E+09	4.400E+06	2.850E+01
4	5	2.200E+01	1.040E+10	4.140E+06	2.850E+01
5	6	3.080E+01	1.203E+10	3.900E+06	2.850E+01
6	7	4.220E+01	1.340E+10	3.730E+06	2.850E+01
7	8	5.250E+01	1.428E+10	3.570E+06	2.850E+01
8	9	6.180E+01	1.482E+10	3.460E+06	2.850E+01
9	10	7.150E+01	1.500E+10	3.400E+06	2.850E+01
10	11	7.420E+01	1.499E+10	3.450E+06	2.850E+01
11	12	7.180E+01	1.476E+10	3.630E+06	2.850E+01
12	13	6.580E+01	1.426E+10	3.920E+06	2.850E+01
13	14	5.880E+01	1.358E+10	4.320E+06	2.850E+01
14	15	5.800E+01	1.254E+10	4.640E+06	2.850E+01
15	16	5.460E+01	1.130E+10	4.600E+06	2.850E+01
16	17	3.830E+01	9.840E+09	3.930E+06	2.850E+01
17	18	2.460E+01	8.100E+09	3.670E+06	2.850E+01
18	19	1.500E+01	5.969E+09	3.070E+06	2.850E+01
19	20	7.000E+00	4.560E+09	2.320E+06	2.850E+01

APPLIED FORCES BY STATIONS

TIME	0	1	2	3	4	5	6
0.00	0.	0.	0.	0.	0.	0.	0.
.01	1.803E+01	1.803E+01	0.	0.	0.	0.	0.
.02	5.065E+01	5.065E+01	0.	0.	0.	0.	0.
.03	8.970E+01	8.970E+01	0.	0.	0.	0.	0.
.04	1.291E+02	1.291E+02	0.	0.	0.	0.	0.
.05	1.707E+02	1.707E+02	0.	0.	0.	0.	0.
.06	1.369E+02	2.153E+02	1.803E+01	0.	0.	0.	0.
.07	2.066E+02	2. 72E+02	5.065E+01	0.	0.	0.	0.
.08	2.109E+02	2.935E+02	8.970E+01	0.	0.	0.	0.
.09	2.126E+02	3.417E+02	1.291E+02	0.	0.	0.	0.
.10	2.131E+02	3.837E+02	1.707E+02	0.	0.	0.	0.
.11	1.951E+02	4.220E+02	2.270E+02	0.	0.	0.	0.
.12	1.624E+02	4.471E+02	2.867E+02	0.	0.	0.	0.
.13	1.244E+02	4.704E+02	3.460E+02	0.	0.	0.	0.
.14	8.400E+01	4.931E+02	4.091E+02	0.	0.	0.	0.
.15	4.240E+01	5.157E+02	4.734E+02	0.	0.	0.	0.
.16	1.515E+01	5.097E+02	5.237E+02	3.007E+01	0.	0.	0.
.17	6.490E+00	4.864E+02	5.580E+02	8.015E+01	0.	0.	0.
.18	2.130E+00	4.531E+02	5.861E+02	1.349E+02	0.	0.	0.
.19	4.700E-01	4.157E+02	6.117E+02	1.964E+02	0.	0.	0.
.20	6.000E-02	3.757E+02	6.360E+02	2.604E+02	0.	0.	0.
.21	0.	3.193E+02	6.663E+02	3.465E+02	0.	0.	0.
.22	0.	2.621E+02	6.943E+02	4.322E+02	0.	0.	0.
.23	0.	2.010E+02	7.185E+02	5.175E+02	0.	0.	0.
.24	0.	1.379E+02	7.418E+02	6.040E+02	0.	0.	0.
.25	0.	7.355E+01	7.640E+02	6.905E+02	0.	0.	0.
.26	0.	3.530E+01	7.393E+02	7.520E+02	4.796E+01	0.	0.
.27	0.	1.635E+01	6.933E+02	7.917E+02	1.147E+02	0.	0.
.28	0.	7.100E+00	6.381E+02	8.220E+02	1.910E+02	0.	0.
.29	0.	2.590E+00	5.781E+02	8.480E+02	2.725E+02	0.	0.
.30	0.	5.200E-01	5.140E+02	8.707E+02	3.572E+02	0.	0.
.31	0.	6.000E-02	4.283E+02	8.940E+02	4.657E+02	0.	0.
.32	0.	0.	3.425E+02	9.155E+02	5.730E+02	0.	0.
.33	0.	0.	2.575E+02	9.365E+02	6.790E+02	0.	0.
.34	0.	0.	1.719E+02	9.545E+02	7.835E+02	0.	0.
.35	0.	0.	8.625E+01	9.717E+02	8.875E+02	0.	0.
.36	0.	0.	4.080E+01	9.268E+02	9.515E+02	6.555E+01	0.
.37	0.	0.	1.974E+01	8.587E+02	9.910E+02	1.520E+02	0.
.38	0.	0.	8.650E+00	7.837E+02	1.022E+03	2.469E+02	0.
.39	0.	0.	3.030E+00	7.050E+02	1.047E+03	3.454E+02	0.
.40	0.	0.	7.000E-01	6.232E+02	1.070E+03	4.472E+02	0.
.41	0.	0.	6.000E-02	5.160E+02	1.064E+03	5.485E+02	0.
.42	0.	0.	0.	4.096E+02	1.040E+03	6.300E+02	0.
.43	0.	0.	0.	3.037E+02	1.016E+03	7.120E+02	0.
.44	0.	0.	0.	1.996E+02	9.940E+02	7.965E+02	0.
.45	0.	0.	0.	9.545E+01	9.710E+02	8.755E+02	0.
.46	0.	0.	0.	4.882E+01	8.923E+02	8.937E+02	5.015E+01
.47	0.	0.	0.	2.935E+01	8.008E+02	8.890E+02	1.176E+02
.48	0.	0.	0.	1.676E+01	7.038E+02	8.743E+02	1.873E+02
.49	0.	0.	0.	8.280E+00	6.037E+02	8.563E+02	2.609E+02
.50	0.	0.	0.	3.230E+00	5.019E+02	8.356E+02	3.369E+02
.51	0.	0.	0.	1.110E+00	4.074E+02	7.746E+02	3.683E+02
.52	0.	0.	0.	1.100E-01	3.215E+02	7.029E+02	3.815E+02
.53	0.	0.	0.	1.000E-02	2.410E+02	6.330E+02	3.920E+02
.54	0.	0.	0.	0.	1.587E+02	5.579E+02	3.990E+02
.55	0.	0.	0.	0.	7.805E+01	4.820E+02	4.040E+02
.56	0.	0.	0.	0.	4.454E+01	4.020E+02	3.575E+02
.57	0.	0.	0.	0.	3.027E+01	3.273E+02	2.970E+02
.58	0.	0.	0.	0.	1.974E+01	2.440E+02	2.242E+02
.59	0.	0.	0.	0.	1.150E+01	1.630E+02	1.505E+02
.60	0.	0.	0.	0.	7.740E+00	8.254E+01	7.400E+01
.61	0.	0.	0.	0.	4.070E+00	4.749E+01	4.342E+01
.62	0.	0.	0.	0.	2.180E+00	3.238E+01	3.016E+01
.63	0.	0.	0.	0.	6.600E-01	2.030E+01	1.972E+01
.64	0.	0.	0.	0.	3.200E-01	1.283E+01	1.250E+01
.65	0.	0.	0.	0.	0.	7.740E+00	7.740E+00
.66	0.	0.	0.	0.	0.	4.070E+00	4.070E+00
.67	0.	0.	0.	0.	0.	2.180E+00	2.180E+00
.68	0.	0.	0.	0.	0.	6.600E-01	6.600E-01
.69	0.	0.	0.	0.	0.	3.000E-01	3.000E-01
5.00	0.	0.	0.	0.	0.	0.	0.

Figure 3 (Continued)

LIST OUTPUT FOR STATIONS:
0 5 7 10 15 20

OPTIONS:	INPUT	FREQS	MODES	TRANSIENT
	1	1	1	1

NATURAL CIRCULAR FREQUENCIES IN RADIANS/SEC
9.465E+00 1.897E+01 2.928E+01

NATURAL FREQUENCIES IN HERTZ
1.506E+00 3.018E+00 4.660E+00

MATRIX OF NATURAL MODES

1.890E+02	-1.194E+02	9.411E+01
1.593E+02	-9.137E+01	6.568E+01
1.291E+02	-6.232E+01	3.559E+01
9.063E+01	-3.297E+01	5.669E+00
6.051E+01	-5.140E+00	-2.034E+01
3.955E+01	1.892E+01	-3.016E+01
1.249E+01	3.689E+01	-4.361E+01
-1.161E+01	4.600E+01	-3.360E+01
-3.160E+01	4.394E+01	-9.612E+00
-4.555E+01	3.029E+01	1.919E+01
-5.232E+01	8.083E+00	3.766E+01
-5.096E+01	-1.612E+01	3.466E+01
-4.175E+01	-3.460E+01	1.287E+01
-2.592E+01	-4.236E+01	-1.412E+01
-5.188E+00	-3.820E+01	-3.271E+01
1.921E+01	-2.340E+01	-3.579E+01
4.502E+01	-1.902E-01	-2.200E+01
7.306E+01	2.888E+01	5.673E+00
1.018E+02	5.925E+01	3.061E+01
1.293E+02	8.899E+01	7.153E+01
1.561E+02	1.167E+02	1.008E+02

				BENDING MOMENT		BENDING STRESS			SHEAR STRESS	
STA	DISPL	VEL	ACCEL	FWD	AFT	FWD	AFT	SHEAR FOR	FWD	AFT
				.02 SECONDS						
0	1.865E-04	-5.057E-03	3.569E+00							
5	-1.549E-05	4.812E-04	-2.850E-01	1.257E+01	1.559E+01	1.030E+00	1.108E+00	1.170E-01	3.250E-01	3.450E-01
7	-4.673E-05	1.296E-03	-8.891E-01	1.643E+01	1.405E+01	1.048E+00	8.412E-01	-9.060E-02	-2.793E-01	-2.918E-01
10	-3.667E-06	3.077E-05	-8.304E-02	2.467E+00	-2.105E+00	1.406E-01	-1.202E-01	-1.742E-01	-5.892E-01	-5.807E-01
15	-1.866E-07	5.562E-05	5.920E-03	4.005E+00	4.623E+00	2.781E-01	3.498E-01	2.355E-02	5.836E-02	5.887E-02
20	6.343E-06	-1.543E-03	1.055E+00							
				.04 SECONDS						
0	1.831E-03	-2.829E-01	8.395E+00							
5	-1.449E-04	2.319E-02	-5.675E-01	4.143E+02	5.165E+02	3.406E+01	3.671E+01	3.891E+00	1.081E+01	1.147E+01
7	-4.556E-04	7.075E-02	-2.043E+00	5.450E+02	4.677E+02	3.477E+01	2.800E+01	-2.945E+00	-9.080E+00	-9.487E+00
10	-4.414E-05	5.909E-03	-3.106E-01	6.677E+01	-6.491E+01	4.946E+00	-3.705E+00	-5.778E+00	-1.954E+01	-1.926E+01
15	4.159E-06	2.411E-05	9.832E-02	1.300E+02	1.503E+02	8.862E+00	1.137E+01	7.732E-01	1.916E+00	1.933E+00
20	2.101E-04	-8.397E-02	2.422E+00							
				.06 SECONDS						
0	6.808E-03	-1.379E+00	1.212E+01							
5	-4.931E-04	1.045E-01	-5.086E-01	2.198E+03	2.733E+03	1.799E+02	1.942E+02	2.076E+01	5.767E+01	6.122E+01
7	-1.672E-03	3.409E-01	-2.802E+00	2.894E+03	2.501E+03	1.847E+02	1.498E+02	-1.497E+01	-4.615E+01	-4.821E+01
10	-2.151E-04	3.837E-02	-7.919E-01	5.152E+02	-2.898E+02	2.937E+01	-1.654E+01	-3.067E+01	-1.037E+02	-1.022E+02
15	5.267E-05	-6.906E-03	3.877E-01	6.588E+02	7.642E+02	4.492E+01	5.782E+01	4.014E+00	9.948E+00	1.008E+01
20	1.113E-03	-4.042E-01	3.362E+00							
				.08 SECONDS						
0	1.656E-02	-3.796E+00	1.332E+01							
5	-1.030E-03	2.530E-01	7.915E-02	6.227E+03	7.803E+03	5.119E+02	5.546E+02	6.005E+01	1.668E+02	1.771E+02
7	-3.987E-03	9.222E-01	-2.765E+00	8.311E+03	7.263E+03	5.303E+02	4.349E+02	-3.991E+01	-1.230E+02	-1.286E+02
10	-7.103E-04	1.443E-01	-1.564E+00	1.729E+03	-5.795E+02	9.853E+01	-3.307E+01	-8.793E+01	-2.974E+02	-2.931E+02
15	2.624E-04	-4.695E-02	9.040E-01	1.753E+03	2.044E+03	1.195E+02	1.546E+02	1.106E+01	2.742E+01	2.765E+01
20	3.191E-03	-1.095E+00	3.533E+00							
				.10 SECONDS						
0	3.160E-02	-7.780E+00	1.268E+01							
5	-1.525E-03	4.186E-01	9.949E-01	1.279E+04	1.611E+04	1.052E+03	1.145E+03	1.264E+02	3.510E+02	3.726E+02
7	-7.395E-03	1.842E+00	-2.163E+00	1.730E+04	1.537E+04	1.104E+03	9.200E+02	-7.370E+01	-2.272E+02	-2.374E+02
10	-1.834E-03	4.055E-01	-2.464E+00	4.165E+03	-4.233E+02	2.488E+02	-2.416E+01	-1.824E+02	-6.170E+02	-6.080E+02
15	8.375E-04	-1.741E-01	1.513E+00	3.253E+03	3.815E+03	2.218E+02	7.886E+02	2.141E+01	5.307E+01	5.353E+01
20	6.662E-03	-2.208E+00	3.378E+00							
				.12 SECONDS						
0	5.163E-02	-1.333E+01	9.212E+00							
5	-1.620E-03	5.031E-01	2.160E+00	2.147E+04	2.722E+04	1.765E+03	1.935E+03	2.189E+02	6.081E+02	6.455E+02
7	-1.165E-02	3.049E+00	-8.058E-01	2.956E+04	2.681E+04	1.886E+03	1.605E+03	-1.050E+02	-3.237E+02	-3.382E+02
10	-3.942E-03	9.265E-01	-3.275E+00	9.182E+03	1.051E+03	5.233E+02	5.996E+01	-3.097E+02	-1.048E+03	-1.032E+03
15	2.016E-03	-4.588E-01	2.034E+00	4.735E+03	5.588E+03	3.229E+02	4.229E+02	3.240E+01	8.050E+01	8.120E+01
20	1.132E-02	-3.732E+00	2.853E+00							
				.14 SECONDS						
0	7.532E-02	-2.014E+01	4.309E+00							
5	-8.572E-04	3.757E-01	3.190E+00	3.131E+04	4.002E+04	2.574E+03	2.844E+03	3.317E+02	9.215E+02	9.782E+02
7	-1.622E-02	4.412E+00	8.350E-01	4.407E+04	4.097E+04	2.812E+03	2.453E+03	-1.181E+02	-3.640E+02	-3.803E+02
10	-7.351E-03	1.810E+00	-3.714E+00	1.698E+04	4.887E+03	9.620E+02	2.789E+02	-4.568E+02	-1.545E+03	-1.523E+03
15	4.030E-03	-9.685E-01	2.233E+00	5.700E+03	6.741E+03	3.886E+02	5.100E+02	3.966E+01	9.830E+01	9.915E+01
20	1.763E-02	-5.622E+00	2.642E+00							
				.16 SECONDS						
0	1.003E-01	-2.766E+01	5.652E-02							
5	1.178E-03	-9.671E-02	3.708E+00	4.084E+04	5.274E+04	3.357E+03	3.748E+03	4.533E+02	1.259E+03	1.337E+03
7	-2.046E-02	5.740E+00	2.138E+00	5.986E+04	5.654E+04	3.769E+03	3.386E+03	-9.592E+01	-2.957E+02	-3.090E+02
10	-1.223E-02	3.126E+00	-3.600E+00	2.784E+04	1.206E+04	1.587E+03	6.884E+02	-6.013E+02	-2.034E+03	-2.004E+03
15	6.864E-03	-1.741E+00	1.954E+00	5.752E+03	6.710E+03	3.922E+02	5.077E+02	3.647E+01	9.039E+01	9.110E+01
20	2.482E-02	-7.930E+00	3.413E+00							
				.18 SECONDS						
0	1.262E-01	-3.536E+01	-2.896E+00							
5	4.675E-03	-9.977E-01	3.568E+00	4.877E+04	6.379E+04	4.010E+03	4.533E+03	5.719E+02	1.589E+03	1.686E+03
7	-2.386E-02	6.659E+00	2.876E+00	7.288E+04	7.214E+04	4.650E+03	4.319E+03	-2.826E+01	-5.714E+01	-9.109E+01
10	-1.854E-02	4.983E+00	-2.874E+00	4.202E+04	2.310E+04	2.395E+03	1.318E+03	-7.209E+02	-2.438E+03	-2.403E+03
15	1.050E-02	-2.764E+00	1.177E+00	4.971E+03	5.395E+03	3.389E+02	4.082E+02	1.616E+01	4.005E+01	4.040E+01
20	3.319E-02	-1.041E+01	5.120E+00							

The mathematical model selected is a finite-element representation of a beam with both bending and shear rigidities, which should give very accurate results. A matrix approach is used so that more complicated hull forms (such as catamarans or surface-effects ships) will require only reprograming the matrix assembly portion to reflect the more complex structural model. Once the mass and stiffness matrices are formed most of the rest of the program can be used unchanged.

The determinant method for eigenvalue extraction has been selected because it is accurate for higher as well as lower modes and because with this method the rigid body modes of the beam do not necessitate preliminary manipulation as some methods do.

The normal mode method allows the user to calculate only the modes of vibration of interest. Rigid body motion is automatically eliminated. If desired, the response in each mode can be printed out separately with a minor modification.

The time-marching method makes it possible to conveniently input a detailed description of the slamming forces with respect to time and length along the ship. The parameter β can be varied to minimize errors in the maximum amplitude or in the period. Table 3 (developed from Reference 10) indicates the tradeoff that has to be made. SLAM uses $\beta = 1/8$. Table 4 (also from Reference 10) indicates the highest frequency for a stable and converging solution.

In the ship slamming problem the participation of the higher modes has many uncertainties associated with it. First the calculated slamming forces are normally derived from a statistical approach. This means that, although a pressure is generally triangular with respect to time and is represented as such, a particular slam may be closer to a half sine, or may be skewed to the left or right, or may have some other shape. This changes considerably the excitation at the higher frequencies, both in magnitude and in phase. The maximum response of the ship will depend on how the higher modes respond relative to the larger fundamental response. For example, if both modal responses peak together, the maximum response would be significantly higher than if the modal responses subtracted from each other. Another complication is that, at least in vibration generator tests on higher modes of ships, the measured mode shapes are sometimes significantly different from the calculated normal mode shapes. Additional measured data, compared with calculations, should help to describe the effect of the higher modes.

TEST PROBLEMS

To test the accuracy of SLAM and to demonstrate its use and characteristics, a series of test problems was solved.

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[10] Newmark, N. M., "Computation of Dynamic Structural Response in the Range Approaching Failure," Symposium on Earthquake and Blast Effects on Structures, University of California at Los Angeles (1952).

TABLE 3 – EFFECT OF FREQUENCY ON ERRORS DUE TO NUMERICAL PROCEDURE IN SLAM

Hz	$\beta = 0$	$\beta = 1/12$	$\beta = 1/8$	$\beta = 1/6$	$\beta = 1/4$
RELATIVE ERRORS IN MAXIMUM RESPONSE TO AN INITIAL VELOCITY					
5	0.012	0.008	0.006	0.004	0
10	0.052	0.034	0.025	0.017	0
20	0.209	0.166	0.116	0.073	0
25	0.614	0.306	0.202	0.122	0
RELATIVE ERRORS IN PERIOD					
5	−0.004	−0.0001	0.002	0.004	0.008
10	−0.017	−0.0003	0.008	0.017	0.033
20	−0.076	−0.006	0.028	0.059	0.121
25	−0.130	−0.015	0.038	0.087	0.179

TABLE 4 – STABILITY AND CONVERGENCE LIMITS FOR β METHOD IN SLAM

	$\beta = 0$	$\beta = 1/12$	$\beta = 1/8$	$\beta = 1/6$	$\beta = 1/4$
Stability Limit in hertz	31.8	38.9	45.0	55.1	∞
Convergence Limit in hertz	∞	55.1	45.0	38.9	31.8

The first series of problems involved a 600-foot uniform steel beam weighing 1000 tons with a bending rigidity, EI, of 10^{10} ton-ft^2. The shear rigidity was assumed to be 10^{21} (very large). The end of the beam was excited laterally by a 10,000-ton step function, and the undamped transient response for the first two modes was calculated. The exact solution for displacement and acceleration of the end of the continuous beam, obtained from Reference 7, is given by the solid lines of Figure 4. The beam was then divided into 20 sections, and solutions were obtained from SLAM which are given by the circles in Figure 4.

In addition to the uniform beam, a MARINER-Class hull was used for some test problems. Figures 5 through 8 show the response of the MARINER hull to slamming forces calculated using SLAM for a slightly more severe than State 6 sea. Frequency and mass dependent damping was used, causing the higher mode components to decay before the fundamental component as shown from experimental results. Figures 5 and 6 show the response, considering only the first three modes. Figures 7 and 8 show the response, considering the first 10 modes. The displacement and accelerations were plotted for the bow and Station 7. The bending moment and shear forces were plotted at Station 7 and amidships.

CONCLUSIONS

Normally a ship hull will be divided into 20 sections for slamming calculations for two reasons: the parameters are likely to be listed at 20 stations for other design studies, and 20 stations is an appropriate number for describing the forces applied to the hull. The accuracy of the solution of the uniform beam (Figure 4) indicates that numerical procedures, using a 20-element representation, are sufficiently accurate for any projected use in connection with conventional ship slamming involving the lower modes. A 10-element model would probably do almost as well in the first three modes; however, the force breakdown may be too coarse.

The user should study Figures 5 through 8 to determine the effect of including higher modes. The displacement and bending moment do not reflect the higher modes as much as the acceleration and shear force. The number of modes requested should be determined by which parameters are important for that particular application and by the accuracy needed. Not much is gained, however, by requesting more modes than half the number of elements.

The flexibility of the analytics used in SLAM allows the user to study intermediate steps in the structural solution: natural frequencies, mode shapes, and individual modal responses. SLAM is flexible also in that it can be easily adapted to handle structural configurations other than a beam.

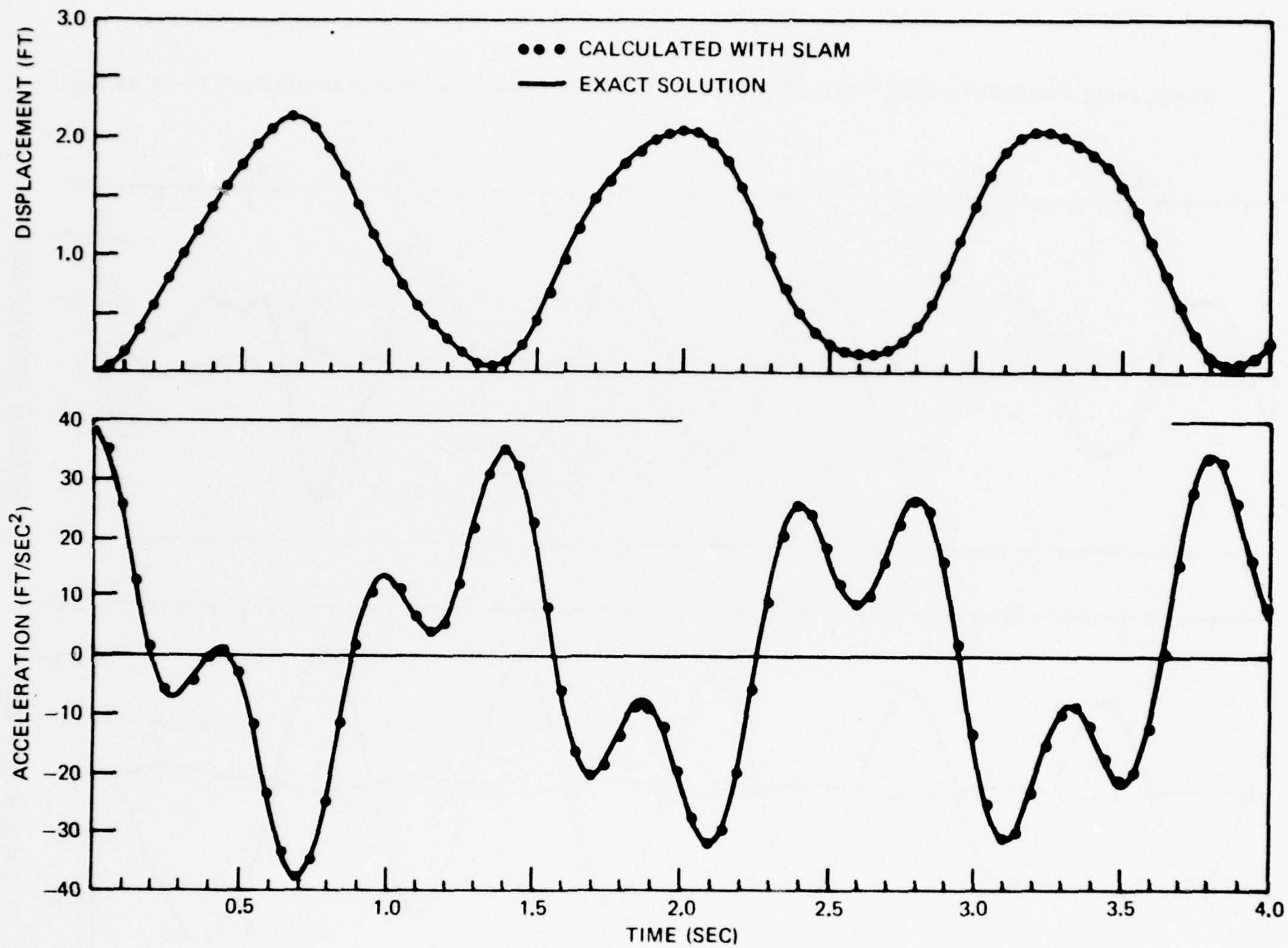


Figure 4 – Displacement and Acceleration Response of the End of a Uniform Euler Beam to a Step Input

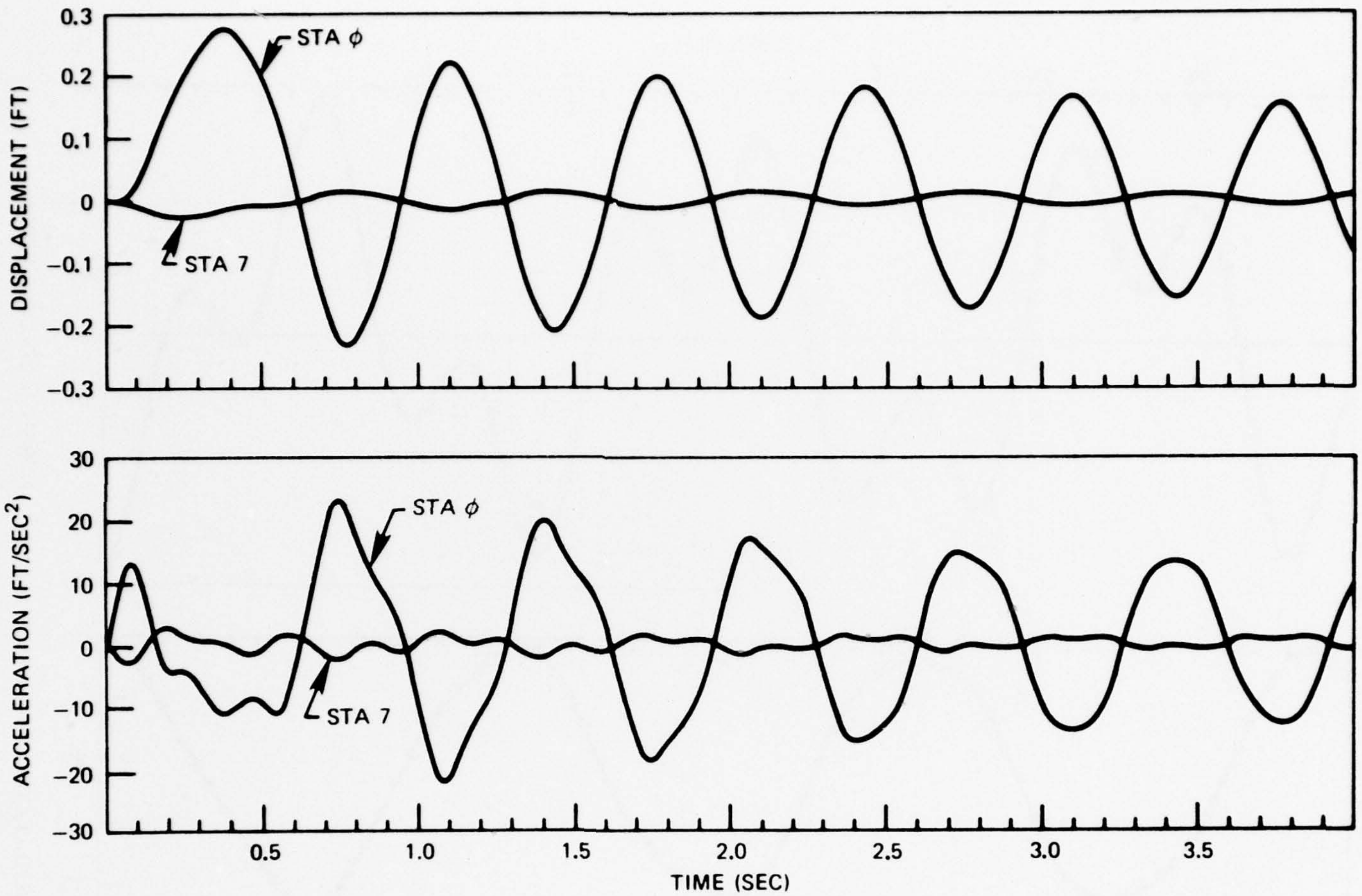


Figure 5 – Displacement and Acceleration Response of a MARINER-Class Hull, Considering Three Modes

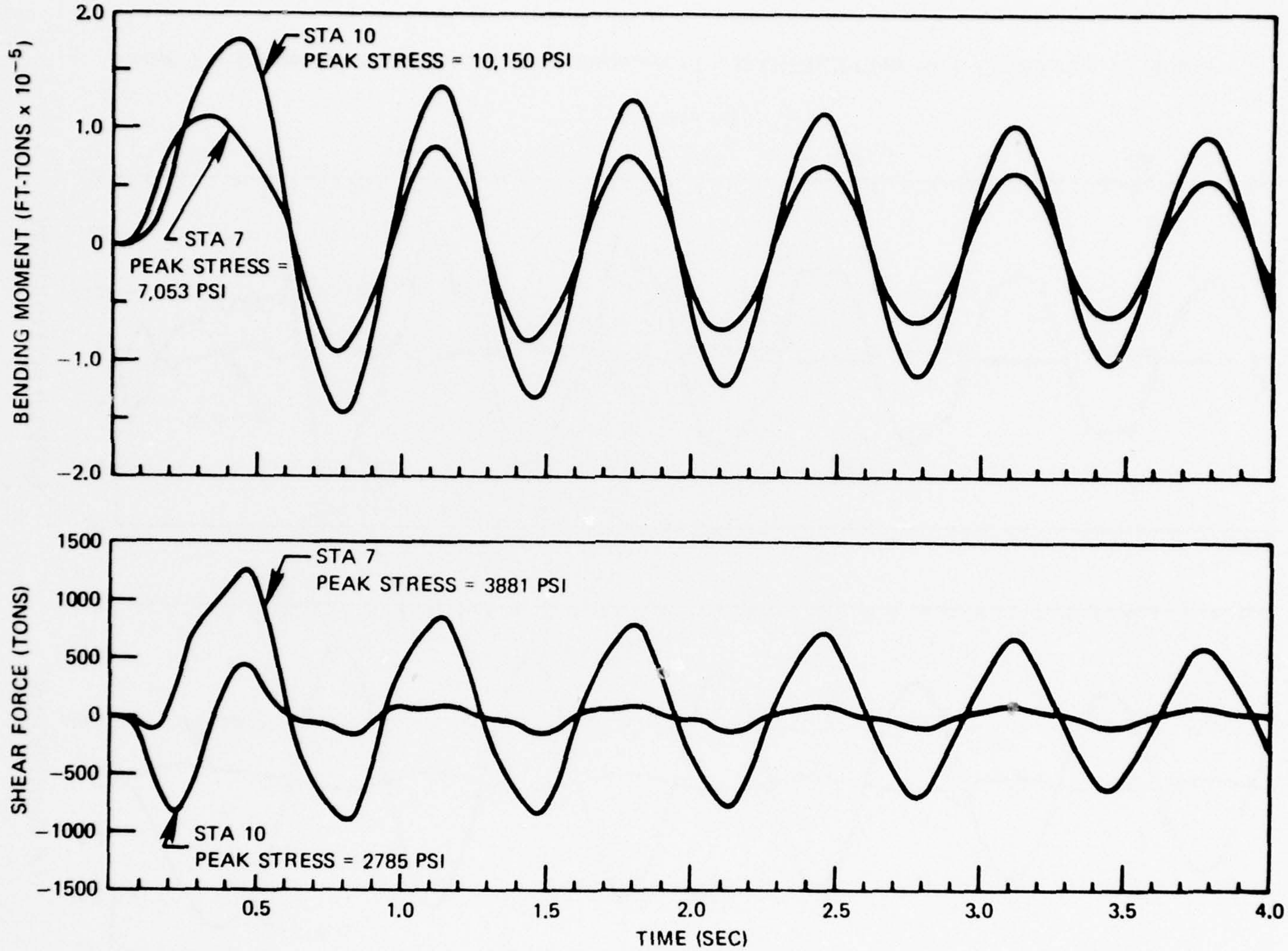


Figure 6 – Bending Moment and Shear Force of a MARINER-Class Hull, Considering Three Modes

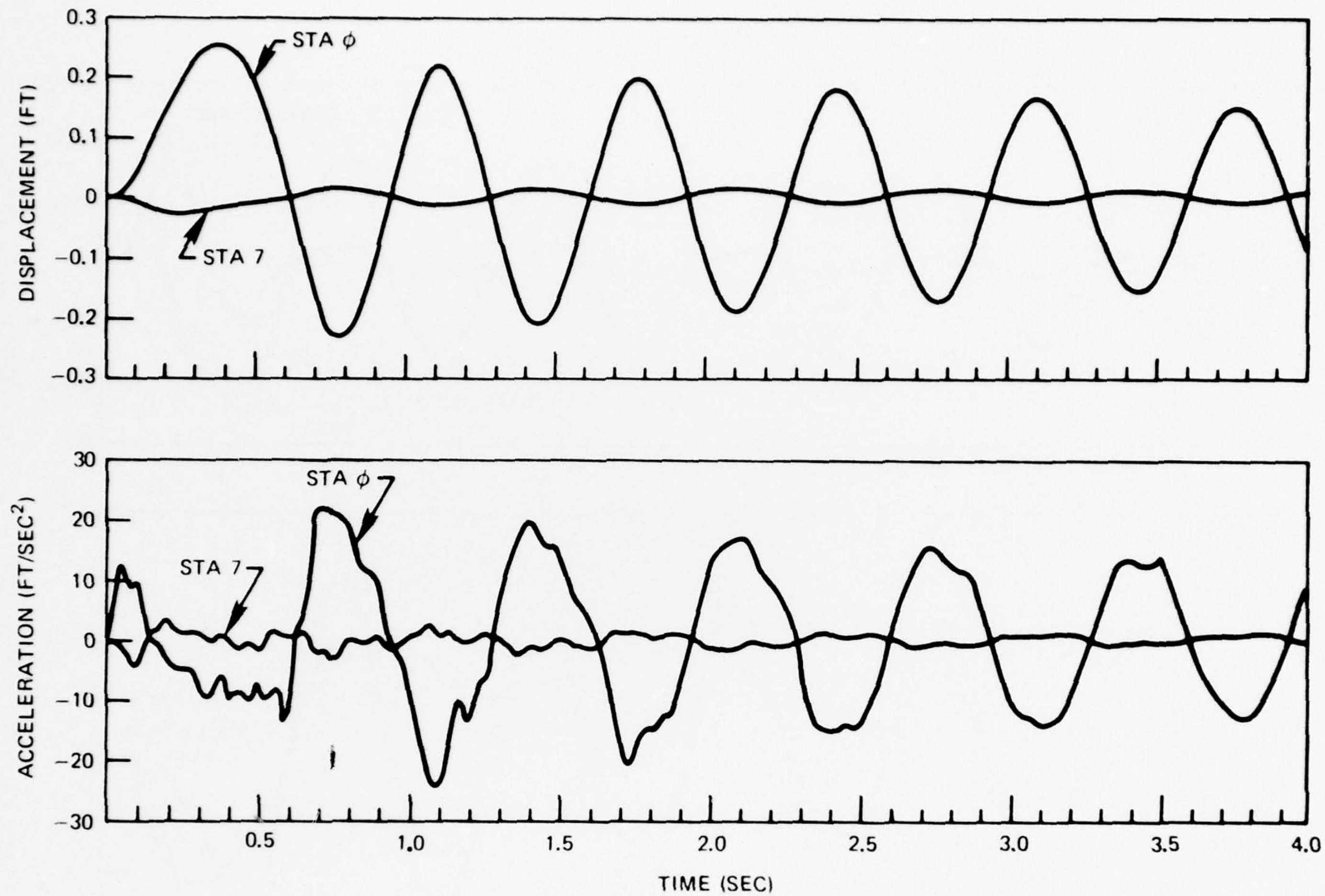


Figure 7 – Displacement and Acceleration Response of a MARINER-Class Hull, Considering 10 Modes

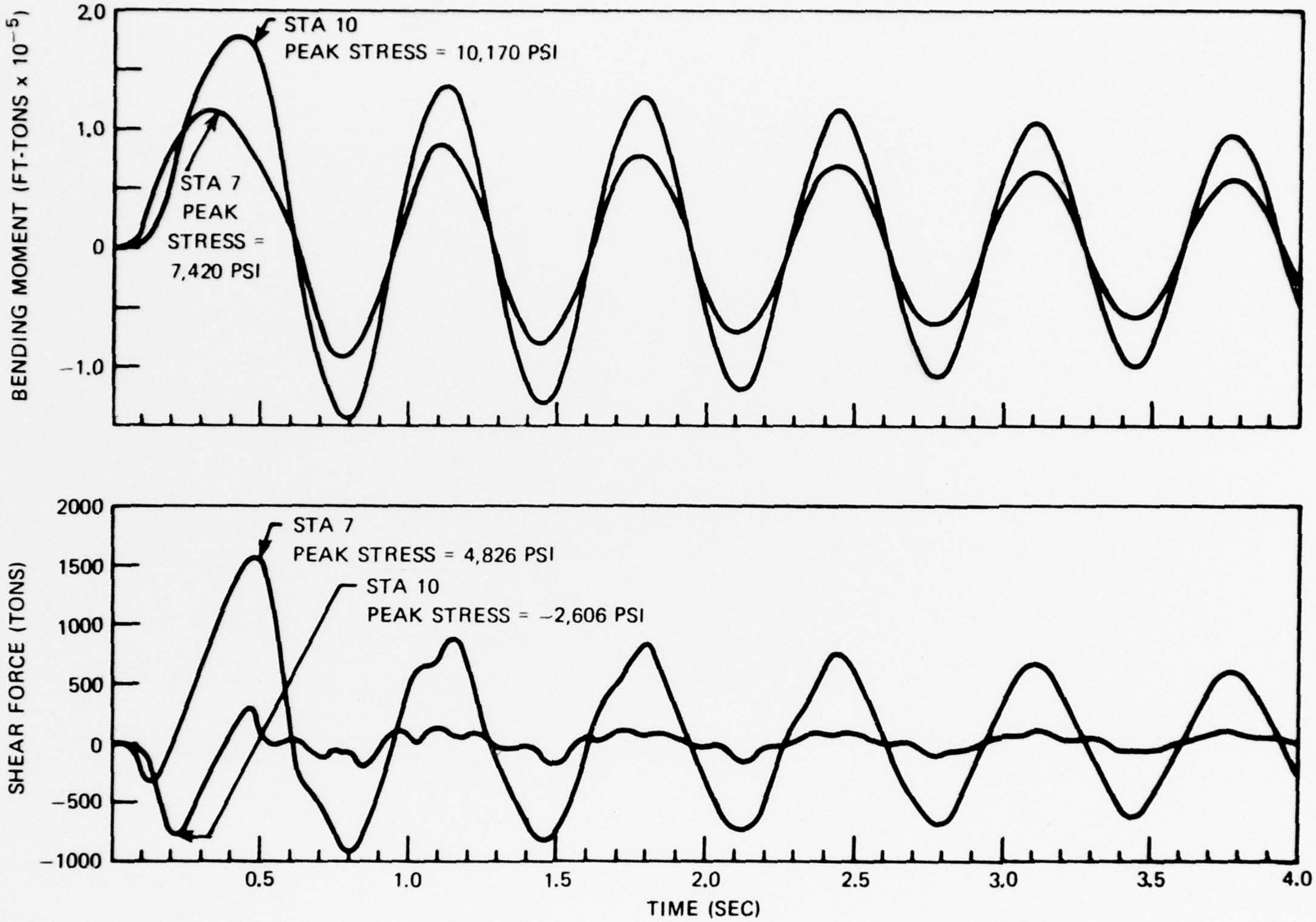
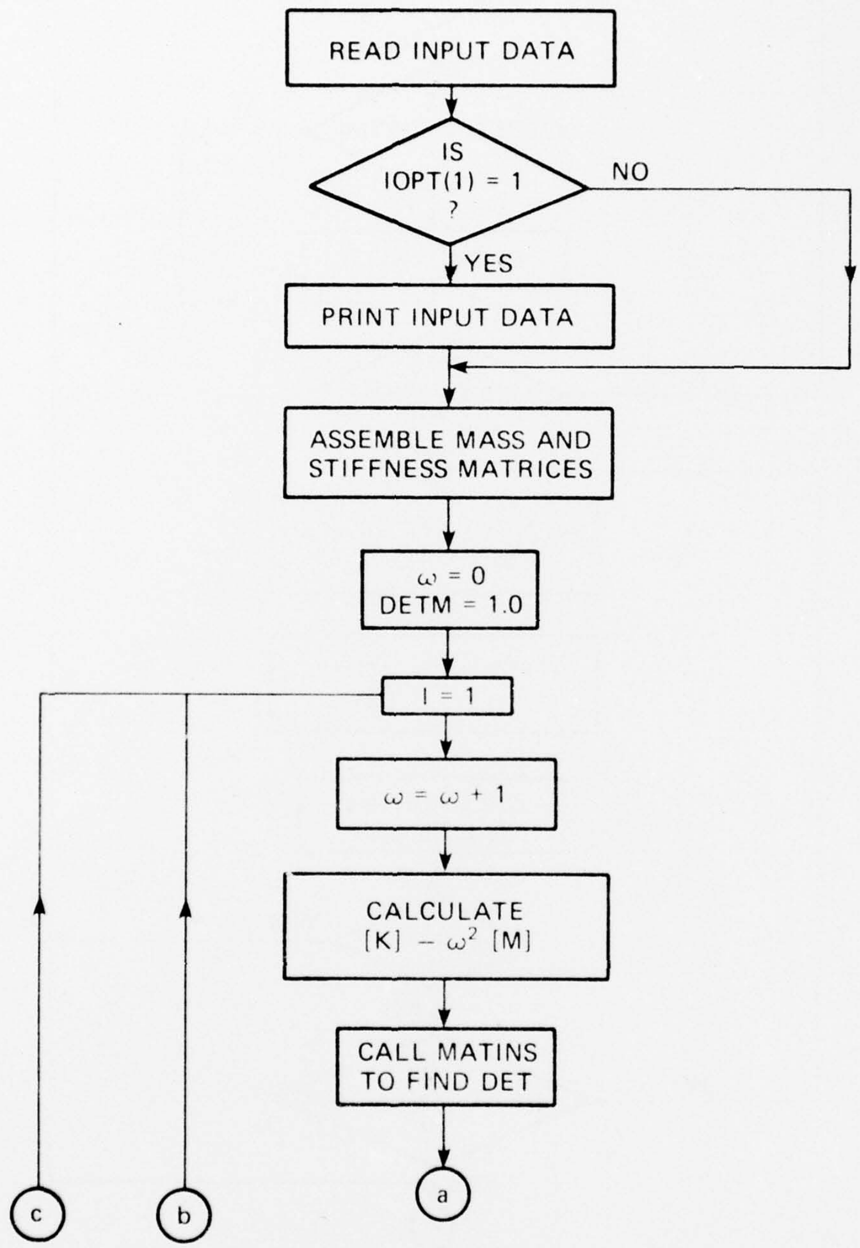
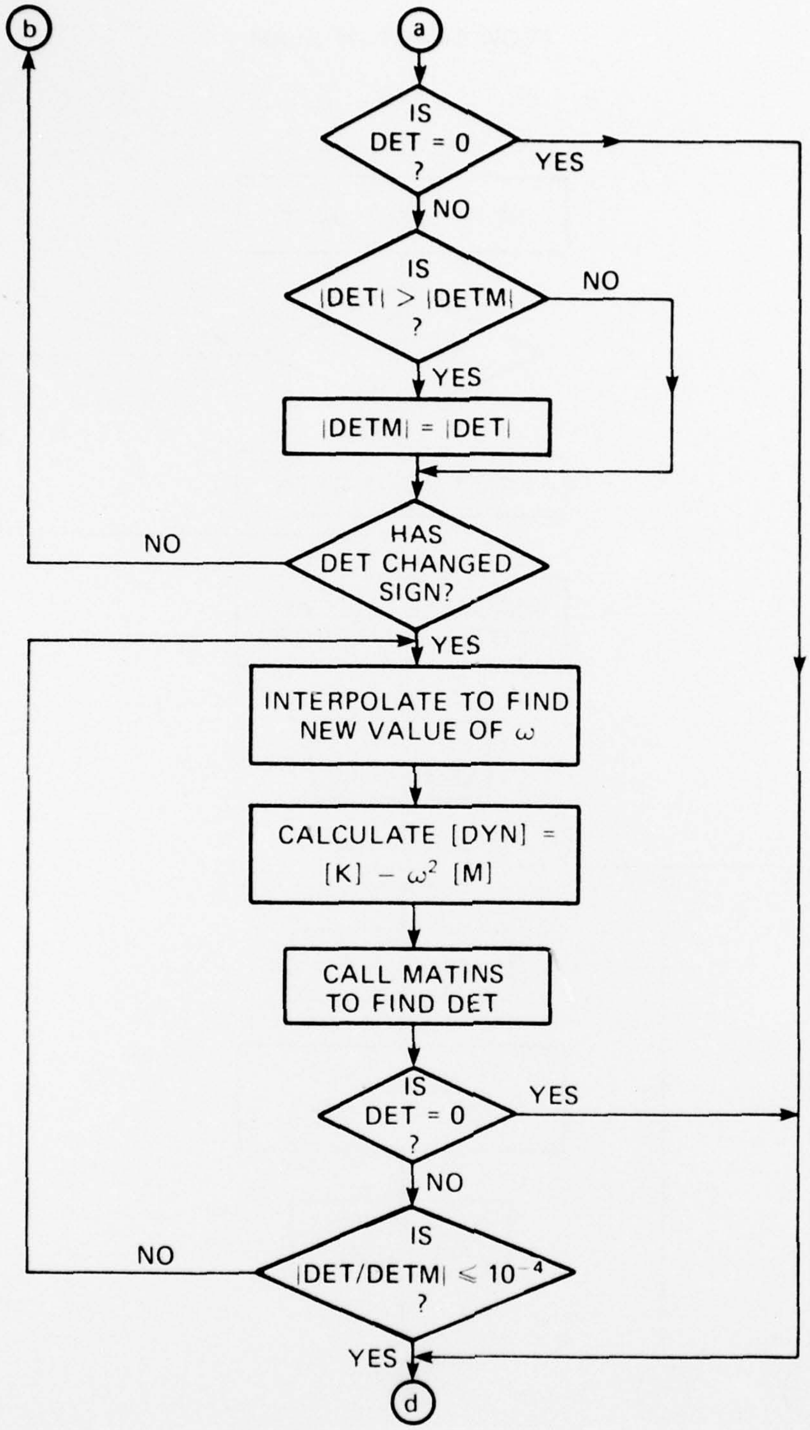


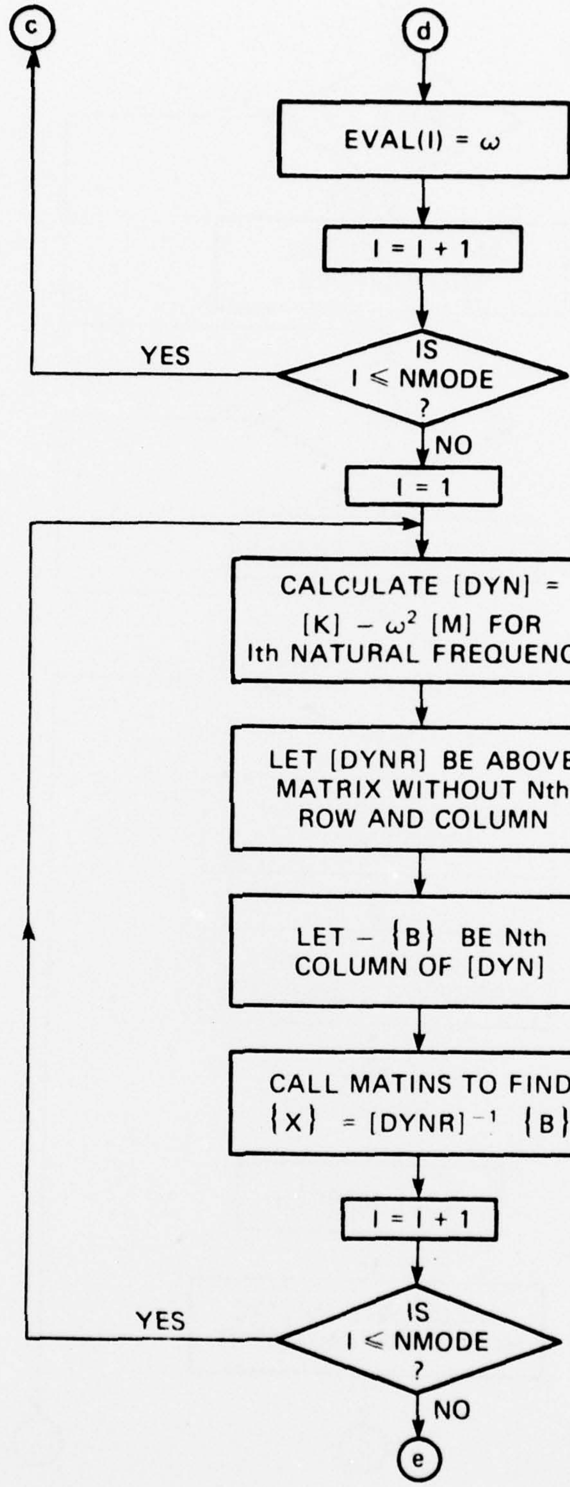
Figure 8 – Bending Moment and Shear Force of a MARINER-Class Hull, Considering 10 Modes

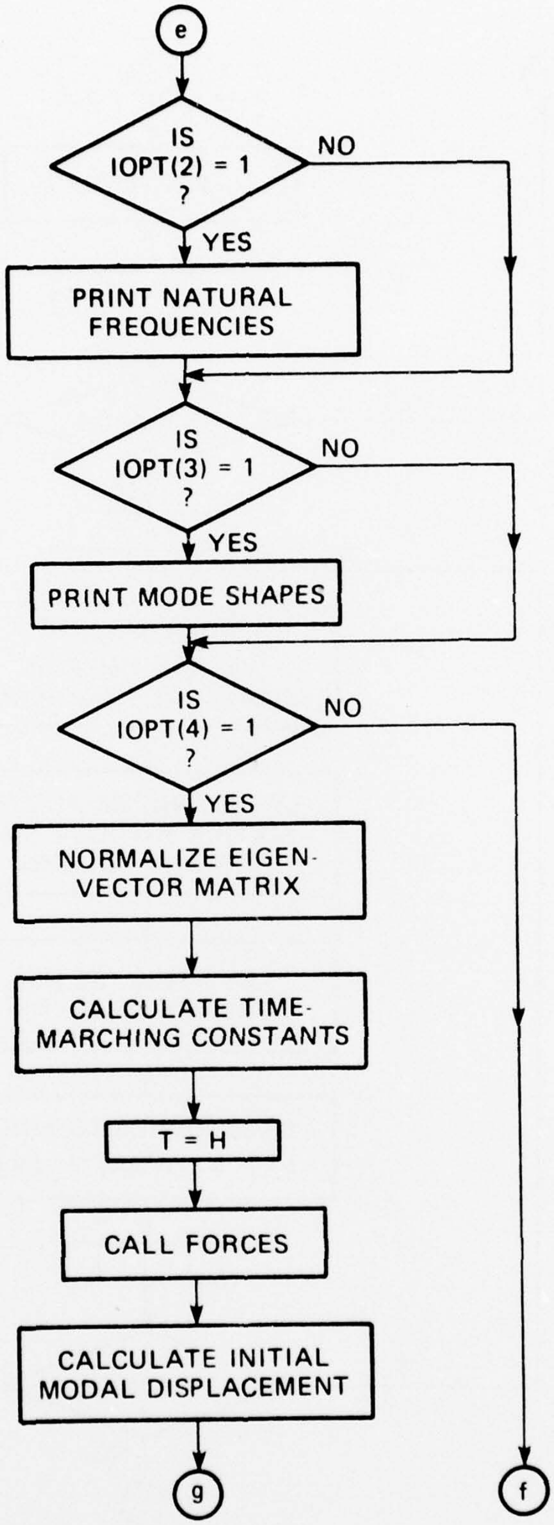
APPENDIX A

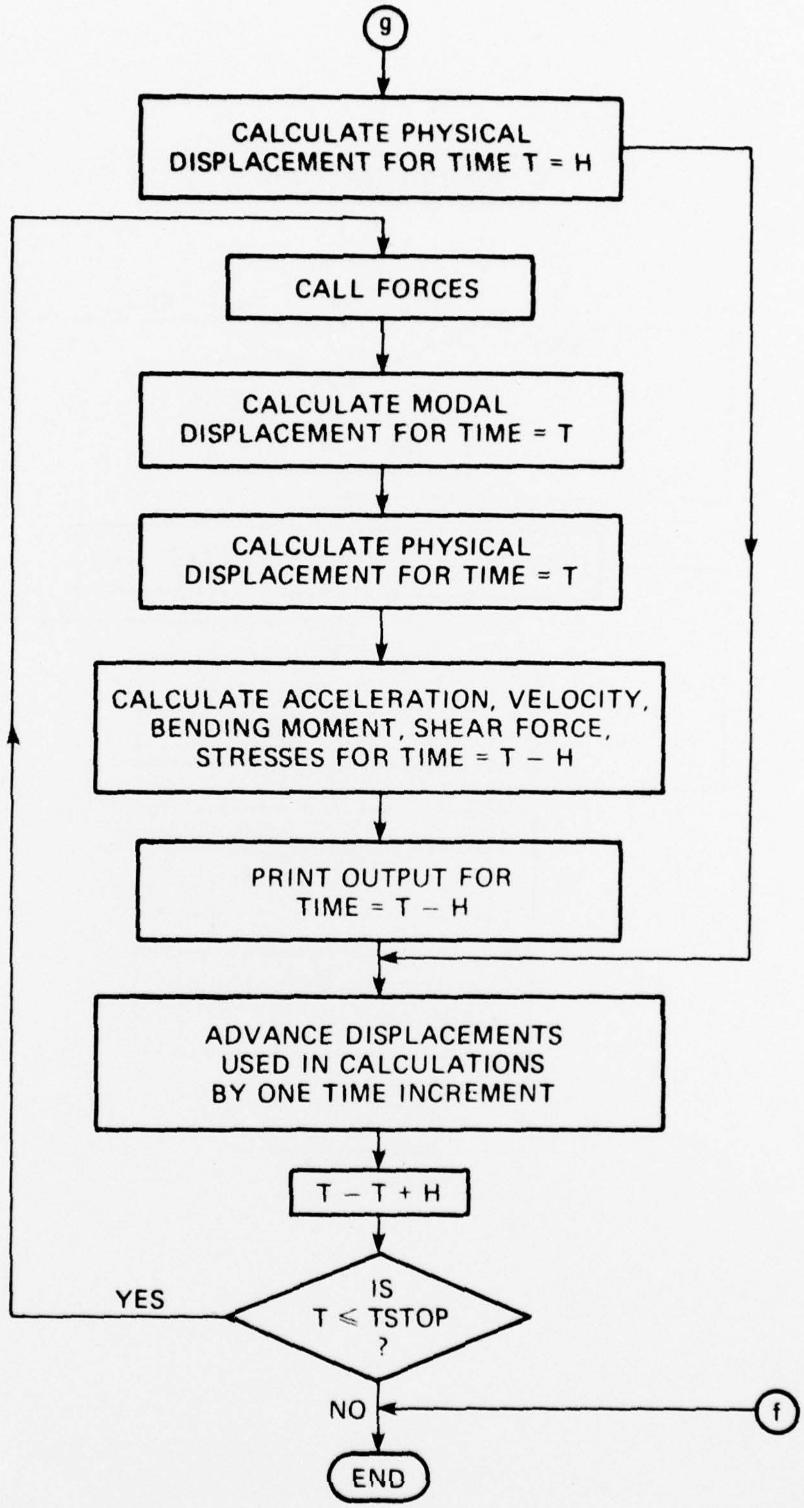
FLOW CHART OF SLAM

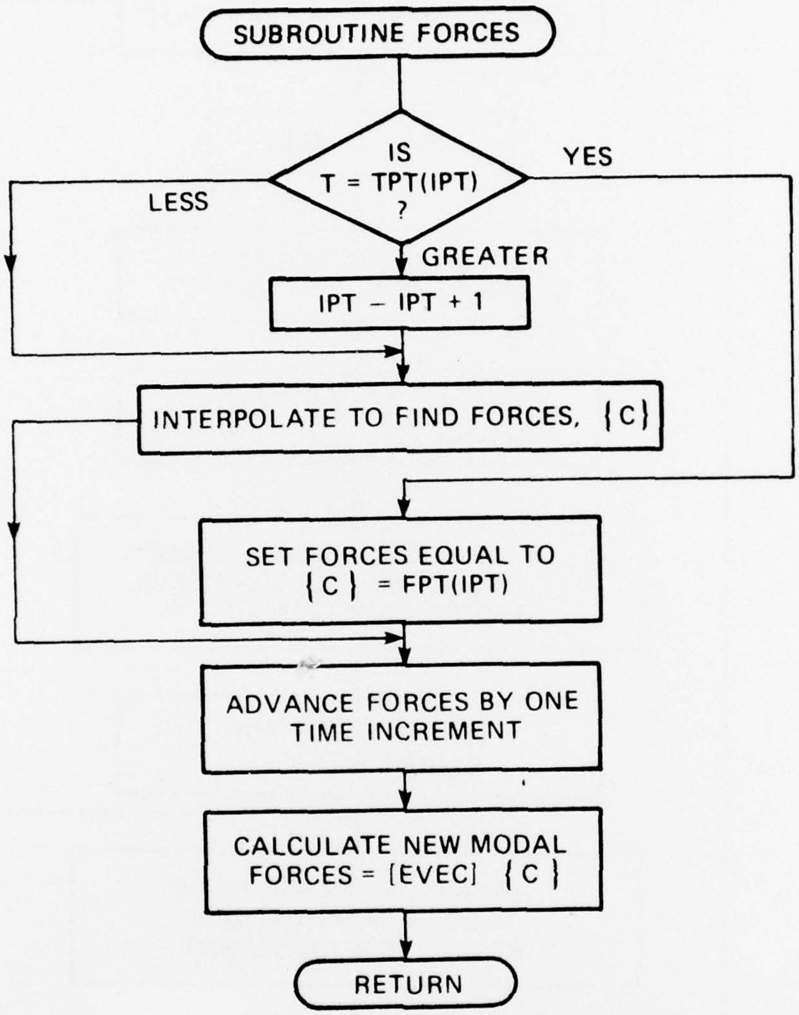



b
a
IS
DET = 0
?
YES
NO
IS
|DET| > |DETM|
?
NO
YES
|DETM| = |DET|
NO
HAS
DET CHANGED
SIGN?
YES
INTERPOLATE TO FIND
NEW VALUE OF ω
CALCULATE [DYN] =
[K] − ω² [M]
CALL MATINS
TO FIND DET
IS
DET = 0
?
YES
NO
NO
IS
|DET/DETM| ≤ 10⁻⁴
?
YES
d


c
d
EVAL(I) = ω
I = I + 1
YES
IS
I ≤ NMODE
?
NO
I = 1
CALCULATE [DYN] =
[K] − ω² [M] FOR
Ith NATURAL FREQUENCY
LET [DYNR] BE ABOVE
MATRIX WITHOUT Nth
ROW AND COLUMN
LET − {B} BE Nth
COLUMN OF [DYN]
CALL MATINS TO FIND
{X} = [DYNR]⁻¹ {B}
I = I + 1
YES
IS
I ≤ NMODE
?
NO
e


e
IS IOPT(2) = 1 ?
NO
YES
PRINT NATURAL FREQUENCIES
IS IOPT(3) = 1 ?
NO
YES
PRINT MODE SHAPES
IS IOPT(4) = 1 ?
NO
YES
NORMALIZE EIGEN-VECTOR MATRIX
CALCULATE TIME-MARCHING CONSTANTS
T = H
CALL FORCES
CALCULATE INITIAL MODAL DISPLACEMENT
g
f


g
CALCULATE PHYSICAL DISPLACEMENT FOR TIME T = H
CALL FORCES
CALCULATE MODAL DISPLACEMENT FOR TIME = T
CALCULATE PHYSICAL DISPLACEMENT FOR TIME = T
CALCULATE ACCELERATION, VELOCITY, BENDING MOMENT, SHEAR FORCE, STRESSES FOR TIME = T – H
PRINT OUTPUT FOR TIME = T – H
ADVANCE DISPLACEMENTS USED IN CALCULATIONS BY ONE TIME INCREMENT
T – T + H
IS T ≤ TSTOP ?
YES
NO
f
END


SUBROUTINE FORCES
IS
T = TPT(IPT)
?
LESS
YES
GREATER
IPT – IPT + 1
INTERPOLATE TO FIND FORCES, {C}
SET FORCES EQUAL TO
{C} = FPT(IPT)
ADVANCE FORCES BY ONE
TIME INCREMENT
CALCULATE NEW MODAL
FORCES = [EVEC] {C}
RETURN

REFERENCES

1. Ochi, M.K. and L.E. Motter, "Prediction of Slamming Characteristics and Hull Responses for Ship Design," Society of Naval Architects and Marine Engineers Transactions (1973).

2. Robinson, D.C., "Calculated Natural Frequencies and Normal Modes of the Guided Missile Cruiser USS LONG BEACH (CG(N)-9)," David Taylor Model Basin Report 2100 (Jan 1966).

3. Perkins, R.L., "Calculated Natrual Frequencies and Normal Modes of Vibration on Range Instrumentation Ship (T-AGM-19)," David Taylor Model Basin Report 1997 (Jun 1965).

4. McGoldrick, R.T., and V.L. Russo, "Hull Vibration Investigation on SS GOPHER MARINER," David Taylor Model Basin Report 1060 (Jul 1956).

5. Antonides, G.P., "A Computer Program for Normal Mode Solutions in Structural Dynamics," Master's Thesis at Catholic University, Washington, D.C. (Dec 1970).

6. MacNeal, Richard H., "The NASTRAN Theoretical Manual (Level 15)," National Aeronautics and Space Administration SP-221(01), Washington, D.C. (Apr 1972).

7. Hurty, W.C. and M.F. Rubinstein, "Dynamics of Structures," Prentice-Hall, Inc., Englewood Cliffs, N.J. (1964).

8. McGoldrick, R.T., "Ship Vibration," David Taylor Model Basin Report 1451 (Dec 1960).

9. Chan, S. et al., "Transient Analysis of Forced Vibrations of Complex Structural-Mechanical Systems," Journal of the Royal Aeronautical Society, Vol. 66, No. 457 (Jul 1962).

10. Newmark, N.M., "Computation of Dynamic Structural Response in the Range Approaching Failure," Symposium on Earthquake and Blast Effects on Structures, University of California at Los Angeles (1952).

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